Integrated Aggregate Production and Marketing Promotion Planning under Uncertainty: A Case Study

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Abstract

This paper provides a general model that optimizes the aggregate production plan and marketing promotion plan simultaneously. There are three types of marketing promotions under consideration which are temporary discount, temporary volume increment, and offering premium gift when some units are bought. The aggregate production plan mainly considers number of workers, overtime, and inventory level in each period. Two main sources of uncertainty, namely, demand and effects of promotions are considered. They are handled by triangular fuzzy numbers, which represent pessimistic, most-likely, and optimistic situations. The optimization model is a fuzzy multi-objective linear programming model. A case study in a real company is used to illustrate the effectiveness of the model. The results show that the model can find a good compromised solution that simultaneously maximize the profits under pessimistic, most-likely, and optimistic situations.

Keywords*: Aggregate production planning, marketing promotion, uncertainty, fuzzy multi-objective linear programming.*

1 Introduction

Aggregate Production Planning (APP) is a medium-term planning over 6 to 18 months. It is used to determine the optimal inventory level, workforce, overtime, and the level of subcontract in order to obtain the optimal solution according to the objective function, e.g., minimizing total cost, maximizing profit, or minimizing the change in workforce level, by taken all manufacturing constraints into account to satisfy the forecasted demand. Marketing Promotion Planning (MPP) is also a medium term plan to determine appropriate levels and types of marketing promotion, e.g., temporary price discount, buy some units get one unit free or get a premium gift, to promote sales and increase the customer demand. Traditionally, aggregate production plan and marketing promotion plan are performed separately, i.e., the marketing promotion activities are planned subjectively and intuitively first in order to increase the customer demands. The objective of the plan is to determine the appropriate promotion activities in each period in order to get the highest sales revenue or the highest profit. In this stage, manufacturing constraints are normally ignored. After that, the demand that is affected by the marketing promotions is forecasted, and then used as an input to determine the aggregate production plan. This traditional approach is not optimal for both marketing and aggregate production plans. In addition, the demand and the effects of promotions are uncertain and not known exactly. They can be estimated under optimistic, most likely, and pessimistic scenarios. There is a risk of making a wrong decision when the real situation is different from the planned situation. For example, if the plan is performed pessimistically and the real situation is optimistic one, the company may lose the sales which results in loss of profit and goodwill. On the other hand, if the plan is performed optimistically and the real situation turns out to be pessimistic, it may result in too high production and inventory carrying costs. These problems lead to the development of an integrated approach of aggregate production planning and marketing promotion which help planning when and what promotion activities should be used, how many units to be sold, while the manufacturing constraints and uncertainties in demand and effects of promotions are taken into account. The uncertainties of demand and effect of marketing promotions are handled by triangular fuzzy numbers that represent the optimistic, most likely, and pessimistic situations. In this paper, a general optimization model is proposed to simultaneously determine the optimal aggregate production, selling, and marketing promotion plans. There are three types of marketing promotions considered in the model which are temporary discount, temporary volume increment, and offering premium gift when some units are bought. The aggregate production plan mainly considers number of workers, overtime, and inventory level in each period. Since the demand and effect of promotion are estimated under optimistic, most likely, and pessimistic situations, the model has three objective functions to simultaneously maximize the total company profits under the three situations. Since the objectives are in conflict, it is impossible to find a solution that gives the real highest profit in all situations at the same time. However, it is possible to find a compromised solution that gets relatively “good” profits in all situations. This paper is organized as follows. The past works are reviewed in the next section. Mathematical model is presented in Section 3, followed by the case study in Section 4. Results are presented and discussed in Section 5 and finally concluded in Section 6.

2 Literature review

This paper is related to aggregate production planning, marketing promotion planning, and decision under uncertainties. Therefore, there are three groups of related past works. There are many research works about the developments and applications of aggregate production planning as follows. *Yenradee, et al.* [1]) and *Yenradee and Piyamanothorn* [2] developed an integrated aggregate production planning with marketing promotion model using linear programming. The model help making decision in marketing promotion in accordance with production planning in order to maximize profit, revenue, or to determine a compromised solution for the company. *Pal, et al.* [3] investigated the effects of partially integrated production and marketing policy (PIPM) of a manufacturing firm which produces single item with a finite rate. The demand of that item is dependent on its selling price, marketing cost and quality. *Buxey* [4] reconciled the theory of aggregate planning for seasonal demand with practical manufacturing. The complex issue of how to disaggregate an optimal aggregate plan never even arises. Managers do not seek perfect solution, but strive to eliminate, or contain, the most significant marginal costs. The nature of the business determines the most appropriate tactics to employ. *Wang and Liang* [5] presented a novel interactive possibilistic linear programming (PLP) approach for solving the multiproduct aggregate production planning (APP) problem with imprecise forecast demand, related operating costs, and capacity. The proposed approach attempts to minimize total costs with reference to inventory levels, labor levels, overtime, subcontracting and backordering levels, and labor, machine and warehouse capacity. *Wang and Fang* [6] presented a fuzzy linear programming method for solving the aggregate production planning problem where the market forecast and the cost of unit subcontract are uncertain in long-term or medium-term production environment. In addition, the specific fuzzy linear programming model is proposed. Moreover, an interactive solution procedure is developed to provide a compromise solution. Research works related to marketing promotion are also studied and summarized as follows. *Alvarez and Casielles* [7] studied the influence of sales promotion to the brand choice behavior. The dependent variable is the brand, and the independent variables are price, reference price, losses and gains, and the different types of sales promotion. They suggested that promotions can have a side effect to consumer that acquiring a brand. It can help to decide which brand to buy when two brands are equally attractive to the consumer. It seems that promotions based on immediate price reductions are the most frequently used and it has been proved that this technique exerts a greater influence on the brand choice process. *Corsten and Gruen* [8] proved that many retailers have been struggling with out-of-stock for long time. They also studied about customers-response when they face the out-of-stock situation. They found that many customers switched brand and never come back. *Smith and Sinha* [9] focused on consumer evaluations of store preference when presented with promotional deals that are equivalent on a unit cost basis and are equivalent on a total cost but are worded differently. An experimental design setting is used to examine the effect of three dial frame. First, state in term of a straight price promotion (50 percent off), the second as an extra product or volume promotion (buy one, get one free) and third is mix promotion (buy two, get 50 percent off). The results suggest that subjects generally prefer promotions which provide immediate gratification with little or no initial investment such as “percent off” or “buy one get one free” in contrast to promotions which appear to require an additional investment such as “buy two, get 50 percent off”. *Ailawadi and Neslin* [10] investigated the effect of sales promotion and established the promotion results in a significant temporal shifting of the demand. They captured the usage rate mechanism by which promotion can increase the demand by modeling consumption during a given period as a function of inventory at the beginning of that period and incorporating this into a jointly estimated purchase incidence and quantity model. *Gupta* [11] examined the impact of promotions on consumer decisions of when, what, and how much to buy. The results indicate that more than 84% of the sales increase due to promotion comes from brand switching (a very small part of which may be switching between different size of the same brand). Purchase acceleration in time accounts for less than 14% of the sales increase. There are many modeling techniques that related to optimization under uncertain environments. One of those is fuzzy linear programming as presented by *Sadeghi and Hosseini* [12]. They proposed that Fuzzy Linear Programming (FLP) is a strategy that can take fuzziness into account. They tried to demonstrate the method of application of FLP for optimization of supply energy system in Iran, as a case study. The FLP model comprises fuzzy coefficients for investment costs.

3 Mathematical model

3.1 *Model characteristics*

The mathematical model in this paper is an extension of traditional aggregate production planning model. In addition to aggregate production plan, it also includes the decisions related to selling and marketing promotion plans. The model also takes the uncertainties in demand and effects of promotions into account using triangular fuzzy numbers representing pessimistic, most likely, and optimistic situations. This model help decide what promotion activities and level of promotions should be used in each period, how many units should be produced, kept in stock, and sold, what strategies should be used to change production rates in each period to get maximum profit in each situation. The model considers single product type but it is very simple to extend it to handle multiple product types. The model can be divided into two parts which are marketing promotion and production planning parts. The marketing promotion part is used to determine the selling plan i.e. the number of units to be sold in each period and the marketing promotion plan i.e. the promotion activity to be used in each period. There are three types of promotion which are temporary discount, temporary volume increment, and temporary offering premium gift when some units are bought. The promotion activities cause the demand to be increased which affect the aggregate production plan. The production planning part is used to determine the optimal inventory level, workforce, overtime and undertime, and the level of subcontract. The objective function of the model is to maximize profit. However, it can be easily changed to any desirable objective of the decision-maker such as maximizing revenue or minimizing cost. Since the demand and effect of promotion are handled by fuzzy numbers representing pessimistic, most likely, and optimistic situations, the profits can be determined under each situations. From the fact that the profits under each situation are conflicting, compromised solutions can be determined.

3.2 *Parameters and variables*

Parameters

T Period (1 to T)

i Promotion type (1 to 3)
- $i = 1$ temporary discount
- $i = 2$ temporary volume increment
- $i = 3$ temporary premium gift when some units are bought

j level of promotion (1 to J)

$n(t)$ Number of working days in period t (days)

$DF(j)$ Discount fraction level for promotion level j

$NB(j)$ Number of units to be bought to get a premium gift for promotion level j (units)

$IF(j)$ Volume increment fraction for promotion level j

$D^{p,m,o}(t)$ Demand in period t (units)

SP Selling price (Baht)

$E^{p,m,o}(i,j)$ Percentage that demand is increased when promotion i level j is used (%)

K Average number of units that can be produced by a worker during regular time in a working day (units)

O_{max} Maximum ratio of overtime hours per regular working hours

C_m Material cost per unit (Baht)

C_p Cost of premium gift (Baht)

C_h Hiring cost (Baht per person)

C_f Firing cost (Baht per person)

C_l Labor cost for regular working hours (Baht per person per day)

C_o Overtime cost (Baht per unit)

C_s Subcontract cost (Baht per unit)

C_i Inventory holding cost (Baht per unit per period)

C_{gw} Loss of goodwill cost per unit of shortage (Baht)

TC Fraction of demand that increased by taking from competitors

RC Factor to be multiplied to reduce material cost when produce at higher volume

Decision Variables

$PC^{p,m,o}(i)$ Total promotion cost for promotion type i during the planning horizon (Baht)

$AD^{p,m,o}(t)$ Adjusted demand in period t, i.e., demand after taking the effect of promotion into account (units)

$FB^{p,m,o}(t)$ Forward buying in period t, i.e., demand that increased by taking the company's own demand in the future (units)

$H(t)$ Number of workers to be hired at the beginning of period t (persons)

$F(t)$ Number of workers to be fired at the beginning of period t (persons)

$W(t)$ Number of workers in period t (persons)

$O(t)$ Overtime production quantity in period t (units)

$U(t)$ Production loss due to idle time in period t (units)

$S(t)$ Subcontract quantity in period t (units)

$I^{p,m,o}(t)$ Inventory level at the end of period t (units)

$IPL(t)$ Planned inventory level at the end of period t (units)

$P(t)$ Total production quantity during period t (units)

$Z(i,j,t)$ 1 if promotion type i level j is used in period t
0 otherwise

$Profit^{p,m,o}$ Total profit during the planning horizon (Baht)

$Revenue^{p,m,o}$ Total sales revenue during the planning horizon (Baht)

RM Cost of material used in production during the planning horizon (Baht)

HF Total hiring and firing cost during the planning horizon (Baht)

$Inv^{p,m,o}$ Total inventory holding cost during the planning horizon (Baht)

LC Total labor cost during the planning horizon (Baht)

$OTcost$ Total overtime cost during the planning horizon (baht)

$SCost$ Total subcontract cost during the planning horizon (Baht)

$LGW^{p,m,o}$ Total loss of goodwill cost during the planning horizon (Baht)

$RMLeft^{p,m,o}$ Cost of material that is used in production but has not been sold (Baht)

$TPC^{p,m,o}$ Total promotion cost during the planning horizon (Baht)

$AS^{p,m,o}(t)$ Actual sales in period t (units)

$PL(t)$ Selling plan in period t (units)

$UD^{p,m,o}(t)$ Unsatisfied demand in period t (units)

$OD^{p,m,o}(t)$ Number of units planned more than demand in period t (units)

$bin^{p,m,o}(t)$ Binary variables to restrict $UD^{p,m,o}(t)$ and $OD^{p,m,o}(t)$ not to be positive at the same time

$PR(t)$ 1 if there is any promotion type used in period t
0 otherwise

$ASP^{p,m,o}(i,j,t)$ Actual sales when promotion type i level j is used in period t (units)

$ASN^{p,m,o}(t)$ Actual sales when no promotion is used in period t (units)

Note that the parameter or variable which contains the superscripts p,m,o is a triangular fuzzy parameter or variable. For example, $D^{p,m,o}(t)$ means $D^{p}(t)$, $D^{m}(t)$, and $D^{o}(t)$ which are demand in pessimistic, most-likely, and optimistic situations, respectively. It can be seen from the list of parameters that there are two fuzzy parameters, namely, the demand, $D^{p,m,o}(t)$ and effect of promotion, $E^{p,m,o}(i,j)$. All other parameters are known constant. From the list of decision variables, there are many fuzzy variables. The values of these variables are uncertain because they are affected by the demand and effect of promotion which are uncertain.

3.3 *Optimization model*

The objective function (1) states that the profit in pessimistic, most likely, and optimistic situations should be maximized at the same time. This means that the model has three objectives, namely, maximizing pessimistic, most likely, and optimistic profits. It can be shown that these objectives are in conflict. The profit in equation (2) is calculated from sales revenue deducted by material cost, hiring and firing cost, inventory holding cost, labor cost, overtime cost, subcontract cost, loss of goodwill cost and total promotion cost. Note that the raw material cost for calculating the profit should be based on the product that is sold out (similar to cost of goods sold in an accounting principle). Equations (3) to (12) are used to calculate the revenue and cost elements required to calculate the profit.

$$Max\ z = Profit^{p,m,o} \tag{1}$$

$$Profit^{p,m,o} = Revenue^{p,m,o} - (RM - RMLeft^{p,m,o}) - HF - Inv^{p,m,o} - LC - OTCost - SCost - LGW^{p,m,o} - TPC^{p,m,o} \tag{2}$$

$$Revenue^{p,m,o} = \sum_{t=1}^{T} AS^{p,m,o}(t) \times SP \tag{3}$$

$$RM = \sum_{t=1}^{T} PL(t) \times C_m \tag{4}$$

$$HF = \sum_{t=1}^{T} (C_h H(t) + C_f F(t)) \tag{5}$$

$$Inv^{p,m,o} = \sum_{t=1}^{T} I^{p,m,o}(t) \times C_i \tag{6}$$

$$LC = \sum_{t=1}^{T} (C_l \times n(t) \times W(t)) \tag{7}$$

$$OTCost = \sum_{t=1}^{T} O(t) \times C_o \tag{8}$$

$$SCost = \sum_{t=1}^{T} S(t) \times C_s \tag{9}$$

$$LGW^{p,m,o} = \sum_{t=1}^{T} UD^{p,m,o}(t) \times C_{gw} \tag{10}$$

$$TPC^{p,m,o} = \sum_{i=1}^{I} PC^{p,m,o}(i) \tag{11}$$

$$RMLeft^{p,m,o} = I^{p,m,o}(t) \times C_m \tag{12}$$

Note that the constraint that contains fuzzy variables or parameters is equivalent to three individual constraints (for pessimistic, most likely, and optimistic situations). For example, constraint (3) is equivalent to constraints (3a, 3b, and 3c). Other constraints in the model are similar.

$$Revenue^{p} = \sum_{t=1}^{T} AS^{p}(t) \times SP \tag{3a}$$

$$Revenue^{m} = \sum_{t=1}^{T} AS^{m}(t) \times SP \tag{3b}$$

$$Revenue^{o} = \sum_{t=1}^{T} AS^{o}(t) \times SP \tag{3c}$$

Marketing Promotion Constraints

Adjusted Demand

$$AD^{p,m,o}(t) = D^{p,m,o}(t) + \sum_{i=1}^{3}\sum_{j=1}^{J}(Z(i,j,t) \times \frac{E^{p,m,o}(i,j)}{100} \times TC \times D^{m}(t)) + FB^{p,m,o}(t) - FB^{p,m,o}(t-1)$$

for t = 1,2,3,…,T (13)

The adjusted demand in each period is a regular demand added by the demand that is increased by using marketing promotions. When the marketing promotion is used, the demand is increased by taking the demand from competitors (the second term on the right side of equation (13)) and taking the company's own demand from the next period called forward buying (the third term on the right side of equation (13)). The marketing promotion in period t-1 brings some parts of the demand from period t to period t-1. Thus, it has negative effect on the demand in period t (the last term on the right side of equation (13)).

Forward-buying

$$FB^{p,m,o}(t) = (1 - TC)\sum_{i=1}^{3}\sum_{j=1}^{J} Z(i,j,t) \times \frac{E^{p,m,o}(i,j)}{100} \times D^{p,m,o}(t+1)$$

for t =1,2,3,…,T (14)

The parameter (TC) is a fraction of the demand that is increased by taking from competitors compared to the total demand that increased when the marketing promotion is used. So, (1-TC) is a fraction of forward-buying.

Selling Plan

$$AD^{p,m,o}(t) - PL(t) = UD^{p,m,o}(t) - OD^{p,m,o}(t) \quad \text{for } t = 1,2,3,\ldots,T \quad (15)$$

$$UD^{p,m,o}(t) \leq bin^{p,m,o}(t) \times BigM \quad \text{for } t = 1,2,3,\ldots,T \quad (16)$$

$$OD^{p,m,o}(t) \leq (1 - bin^{p,m,o}(t)) \times BigM \quad \text{for } t = 1,2,3,\ldots,T \quad (17)$$

Selling plan ($PL(t)$) is a decision variable to determine the quantity planned to be sold in each period. Equation (15) is to determine number of units of demand that is unsatisfied ($UD^{p,m,o}(t)$) and number of units that is planned more than the demand ($OD^{p,m,o}(t)$). Constraints (16) and (17) are to prevent $UD^{p,m,o}(t)$ and $OD^{p,m,o}(t)$ from having positive values at the same time.

Actual Sales

$$ASP^{p,m,o}(i,j,t) \leq Z(i,j,t) \times BigM \quad \text{for } t = 1,2,3,\ldots,T,\ i = 1,2,3 \text{ and } j = 1,2,3,\ldots,J \quad (18)$$

$$ASN^{p,m,o}(t) \leq (1 - PR(t)) \times BigM \quad \text{for } t = 1,2,3,\ldots,T \quad (19)$$

$$PR(t) = \sum_{i=1}^{3}\sum_{j=1}^{J} Z(i,j,t) \quad \text{for } t = 1,2,3,\ldots,T \quad (20)$$

$$AS^{p,m,o}(t) = \sum_{i=1}^{3}\sum_{j=1}^{J} ASP^{p,m,o}(i,j,t) + ASN^{p,m,o}(t) \quad \text{for } t = 1,2,3,\ldots,T \quad (21)$$

$$AS^{p,m,o}(t) = PL(t) - OD^{p,m,o}(t) \quad \text{for } t = 1,2,3,\ldots,T \quad (22)$$

To calculate marketing promotion costs, it needs to divide the actual sales into two types, namely, the actual sales when promotion is used, $ASP^{p,m,o}(i,j,t)$ and actual sales when promotion is not used, $ASN^{p,m,o}(t)$. Constraint (18) allows the actual sale when the promotion is used to be positive when the promotion is used. Similarly, the actual sale when the promotion is not used is allowed to be positive when there is no promotion in that period as shown by constraints (19) and (20). Equation (21) shows that actual sales in each period ($AS^{p,m,o}(t)$) comes from actual sales when a marketing promotion is used ($ASP^{p,m,o}(i,j,t)$) and actual sales when a marketing promotion is not used ($ASN^{p,m,o}(t)$). Equation (22) expresses that the actual sales in each period ($AS^{p,m,o}(t)$) is equal to selling plan deducted by the amount that selling plan is more than the adjusted demand. This means that the actual sale is the minimum between the adjusted demand and the selling plan.

Additional marketing promotion constraints

$$\sum_{i=1}^{3}\sum_{j=1}^{J} Z(i,j,t) \leq 1 \quad \text{for } t = 1,2,3,\ldots,T \quad (23)$$

$$\sum_{j=1}^{J}\sum_{t=1}^{T} Z(1,j,t) \geq 1 \quad (24)$$

$$\sum_{j=1}^{J}\sum_{t=1}^{T} Z(2,j,t) \geq 1 \quad (25)$$

$$\sum_{j=1}^{J}\sum_{t=1}^{T} Z(3,j,t) \geq 1 \quad (26)$$

Equation (23) is a mutually exclusive constraint to restrict that a marketing promotion used in a period can be only one type with one level. Otherwise, it will confuse the customers and will not be practical. Equations (24) to (26) are constraints to force that each type of promotion must be used at least once during the planning horizon. They are required to diversify types of marketing promotion, otherwise only one type of promotion may be used.

Promotion Costs

$$PC^{p,m,o}(1) = \sum_{j=1}^{J}\sum_{t=1}^{T} ASP^{p,m,o}(1,j,t) \times DF(j) \times SP \quad (27)$$

$$PC^{p,m,o}(2) = \sum_{j=1}^{J}\sum_{t=1}^{T} ASP^{p,m,o}(2,j,t) \times IF(j) \times C_m \times RC \quad (28)$$

$$PC^{p,m,o}(3) = \sum_{j=1}^{J}\sum_{t=1}^{T} \frac{ASP^{p,m,o}(3,j,t) \times C_p}{NB(j)} \quad (29)$$

Equations (27) to (29) are used to determine marketing promotion costs. They are calculated based on the actual sales when the promotion is used, $ASP^{p,m,o}(i,j,t)$. The temporary discount promotion cost, $PC^{p,m,o}(1)$, is determined based on the discount fraction and the selling price as shown in equation (27). The promotion cost of temporary volume increment in equation (28) is based on the required additional material cost. When the volume of the product is increased by 50% the material cost may be increased less than 50% because of production economy. Thus, the factor RC is a correction factor provided for this purpose. From equation (29), $ASP^{p,m,o}(3,j,t)/NB(j)$ is a number of units of premium gift given to customers.

Aggregate Production Planning Constraints

Conservation of workforce

$$W(t) = W(t-1) + H(t) - F(t) \quad \text{for } t = 1,2,3,\ldots,t \quad (30)$$

Limitation of overtime production quantity

$$O(t) = O_{max} \times K \times n(t) \times w(t) \quad \text{for } t = 1,2,3,\ldots,t \quad (31)$$

Production level

$$P(t) = K \times n(t) \times w(t) + O(t) - U(t) \quad \text{for } t = 1,2,3,\ldots,t \quad (32)$$

Inventory balance

$$IPL(t) = IPL(t-1) + P(t) + S(t) - PL(t) \quad \text{for } t = 1,2,3,\ldots,t \quad (33)$$

$$I^{p,m,o}(t) = I^{p,m,o}(t-1) + P(t) + S(t) - AS^{p,m,o}(t) \quad \text{for } t = 1,2,3,\ldots,t \quad (34)$$

Equation (33) shows planned inventory in each period ($IPL(t)$) where equation (34) shows real inventory in pessimistic, most likely, and optimistic situations ($I^{p,m,o}(t)$). Note that there is a single selling plan which has a constant value but the actual sales are fuzzy variables dependent on situations.

Non-negativity and integer conditions

$H(t)$ and $F(t)$ are integer for $t = 1,2,3,\ldots,T$

$Z(i,j,t)$ are integer for $i = 1,2,3$

$j = 1,2,3,\ldots,J$

$t = 1,2,3,\ldots,T$

All parameters and variables are non-negative (35)

3.4 *Compromised solution*

The objective function (1) is equivalent to three objectives of simultaneously maximizing pessimistic, most likely, and optimistic profits. It can be transformed to an equivalent single objective model. The objective function (1) is transformed to objective function (36) and constraints (37) and (38). Since the model gives a compromised solution, it is called a compromised model. Note that the original constraints (2) to (35) are still needed.

Compromised model:

$$Max\ z = \alpha \quad (36)$$

Subject to:

$$\alpha \le \alpha^{p,m,o} \quad (37)$$

$$\alpha^{p,m,o} = \frac{Profit^{p,m,o} - Minprofit^{p,m,o}}{Maxprofit^{p,m,o} - Minprofit^{p,m,o}} \quad (38)$$

$\alpha^{p,m,o}$ in constraint (37) is the satisfaction level for pessimistic, most-likely, and optimistic profits respectively. In equation (38), $Profit^{p,m,o}$ is the profit from the compromised solution under pessimistic, most-likely and optimistic situations, respectively. $Minprofit^{p,m,o}$ and $Maxprofit^{p,m,o}$ are minimum profit and maximum profit of all solutions under pessimistic, most-likely, and optimistic situations, respectively. A guideline to determine $Minprofit^{p,m,o}$ and $Maxprofit^{p,m,o}$ will be presented in section 5.2. The compromised model is to maximize minimum of the satisfaction levels for pessimistic, most likely, and optimistic profits. The maximum and minimum possible satisfaction levels are 1.0 and 0.0, respectively. For example, α^{m} will be equal to 1.0 if the $Profit^{m}$ is equal to $Maxprofit^{m}$ while α^{m} will be equal to 0.0 if the $Profit^{m}$ is equal to $Minprofit^{m}$.

4 Case study

A numerical experiment is used to illustrate an effectiveness of the model. It is based on a real situation in a company producing consumer products in Thailand. However, the numerical values of data are adjusted to protect the company's confidentiality. This company produces one product and the regular demand in each period under pessimistic, most likely, and optimistic situations, $D^{p,m,o}(t)$, are presented in Table 3. When the demand is higher than the selling plan, it has an unsatisfied demand. The unsatisfied demand results in a loss of current sales and also loss of goodwill that leads to loss of future sales. There are three types of promotion which are temporary discount, temporary volume increment, and a premium gift when some units are bought. There are four levels of temporary discount, which are 10%, 20%, 30%, and 50%. There are three levels of temporary volume increment, which are 20%, 30%, and 40%. The factor *RC* is 0.8, which means that when the product volume is increased by x%, the material cost is increased by $0.8x$%. There are two levels of the premium gift promotion. The customer gets a premium gift when 2 or 3 units are bought dependent on the level of promotion. The company has a policy to use each type of promotion at least once in a planning horizon of 6 months. The effect of each type and each level of promotion is shown in Table 1. It is estimated that when the promotion is used, 80% of the demand increased is from the competitors, the other 20% is from the company's own demand in the next period. This means that the factor TC is 0.8. The number of working days in each period, $n(t)$, are 20, 24, 24, 18, 26, and 26 days for periods 1 to 6, respectively. Other aggregate production planning data required in the model are shown in Table 2.

Table 1: Effect of promotions, $E^{p,m,o}(i,j)$

Situation	Pessimistic (p)	Most-likely (m)	Optimistic (o)
Temporary discount (i=1)			
10% (j=1)	24%	40%	56%
20% (j=2)	36%	60%	84%
30% (j=3)	48%	80%	112%
50% (j=4)	60%	100%	140%
Temporary volume increment (i=2)			
20% (j=1)	12%	20%	28%
30% (j=2)	18%	30%	42%
40% (j=3)	24%	40%	56%
Premium gift (i=3)			
When 2 units are bought (j=1)	48%	80%	112%
When 3 units are bought (j=2)	36%	60%	84%

Table 2: Production data

Initial inventory, $I^{p,m,o}(0)$	100 units
Initial number of worker, $W(0)$	10 people
Normal sale price, SP	350 Baht per unit
Material cost per unit, C_m	100 Baht per unit
Premium gift cost, C_p	120 Baht per unit
Hiring cost, C_h	2,000 Baht per person
Firing cost, C_f	5,000 Baht per person
Inventory holding cost, C_i	5 Baht per unit per period
Wage of labor, C_l	240 Baht per person per day
Overtime cost, C_o	180 Baht per unit
Subcontract cost, C_s	198 Baht per unit
Loss of goodwill cost, C_{gw}	25 Baht per unit of shortage
Capacity of worker, K	2 units per worker per day
Maximum ratio of overtime hours per regular working hours, O_{max}	0.25

5 Result and discussion

The results are divided into two parts. First, it is assumed that a planner knows exactly in advance what situation among pessimistic, most likely, and optimistic ones will occur. Suppose the pessimistic situation will occur, two fuzzy parameters, namely, the demand, $D^{p,m,o}(t)$, and effect of promotion, $E^{p,m,o}(i,j)$ will have pessimistic values. The optimization model includes formula (1) to (35) where all fuzzy parameters and fuzzy variables have only the pessimistic element. This means that the objective function (1) is to maximize only the pessimistic profit. This model gives a solution called pessimistic solution. Similarly, the most likely and optimistic solutions will be obtained when the fuzzy parameters have most likely and optimistic values, respectively. The most likely solution is presented in Table 3. Note that the pessimistic and optimistic solutions are not presented to save space. Second, it is assumed that the planner cannot know in advance what situation will occur. This case is obviously more practical. The best solution for one situation may be the worst for other situations. For example, the optimistic solution may perform very badly under pessimistic situation, and vice versa. The most likely solution may perform unsatisfactorily under pessimistic or optimistic situation. In this case the planner may prefer a compromised or "good" but not the best solution under any situation. The models are solved using optimization software LINGO 8.0. The computer used is a laptop with CPU AMD® Turion™ 64 X2 TL - 521.6 GHz and 2.5 GB of RAM. The computation time for each model is approximately 1 minute and 20 seconds.

5.1 *The best solutions in each situation*

Table 3 contains very useful information for planners. The information includes optimal types and levels of marketing promotion, regular and adjusted demands under each situation, selling plan and actual sales under each situation, aggregate production plan including inventory levels under each situation. From Table 3 the solution shows that the promotions are used in the periods of relatively low demand. The promotions during low demand periods tend to reduce the degree of seasonality of demand. This results in a more efficient production plan. The profits of each solution under each situation are summarized in Table 4. It can be seen that there is no solution that is the best in all situations, for instances, the pessimistic, most likely, and optimistic solutions are the best under pessimistic, most likely, and optimistic situations, respectively. However, the pessimistic, most likely, and optimistic solutions has the worst performance under optimistic, pessimistic, and pessimistic situations, respectively. The most likely solution has lower profit under optimistic situation because of loss of goodwill cost. It also has less profit under pessimistic solution due to lower actual sales and higher inventory holding cost. It is possible to find the best solution under a given situation but not the best under all situations. Nevertheless, a compromised solution under all situations can be determined.

Table 3: Most-likely solution

		Period					
	Situation	1	2	3	4	5	6
Regular Demand	Pessimistic	640	800	480	960	640	480
	Most-likely	800	1,000	600	1,200	800	600
	Optimistic	960	1,200	720	1,440	960	720
Adjusted Demand	Pessimistic	736	780	480	960	817	676
	Most-likely	800	1,000	600	1,200	800	600
	Optimistic	1,206	1,133	720	1,440	1,399	1,204
Production		868	1,008	1,008	756	1,092	1,092
Overtime		28	0	0	0	0	0
Undertime		0	0	0	0	0	0
Subcontract		0	0	0	0	0	0
No. of workers		21	21	21	21	21	21
No. of hiring		11	0	0	0	0	0
No. of firing		0	0	0	0	0	0
Selling Plan		968	960	600	1,200	1,104	936
Actual Sales	Pessimistic	736	781	480	960	817	676
	Most-likely	968	960	600	1,200	1,104	936
	Optimistic	968	960	600	1,200	1,104	936
Promotion		Volume increment 20%	-	-	-	Discount 10%	Buy 3 units Get premium gift
Inventory	Pessimistic	232	459	987	783	1,059	1,475
	Most-likely	0	48	456	12	0	156
	Optimistic	0	48	456	12	0	156

Table 4: Profits of each solution under each situation

		Real situations			α
		Pessimistic	Most-likely	Optimistic	
Solutions	Pessimistic	\$499,607(α^p =1.0)	\$466,956(α^m =0.27)	\$433,927(α^o =0.0)	0.0
	Most-likely	\$312,993(α^p =0.61)	\$640,112(α^m =1.0)	\$606,760(α^o =0.49)	0.49
	Optimistic	\$22,086(α^p =0.0)	\$402,017(α^m =0.0)	\$785,366(α^o =1.0)	0.0

5.2 *Compromised solutions*

When a compromised solution is needed, a simultaneous optimization technique using fuzzy programming approach can be used as explained in section 3.4. Equation (38) requires the values of $\text{Minprofit}^{p,m,o}$ and $\text{Maxprofit}^{p,m,o}$. Their values are obtained from Table 4. The Minprofit^{p} is the minimum in pessimistic column which is \$22,086 while Maxprofit^{p} is the maximum in pessimistic column which is \$499,607. Similarly, Minprofit^{m}, Maxprofit^{m}, Minprofit^{o}, and Maxprofit^{o} are \$402,017, \$640,112, \$433,927, and \$785,366, respectively. The satisfaction levels, α, of the pessimistic, most likely, and optimistic solutions are calculated and presented in Table 4. They are 0.0, 0.49, and 0.0, respectively. This means that the most likely solution is better since the satisfaction level is higher. When the compromised model including formula (36) to (38) and (2) to (35) is solved, the satisfaction level, α, is 0.58 and the profits under each situation are presented as compromised solution 1 in Table 5. It can be seen from Tables 4 and 5 that the compromised solution 1 is more similar to the most likely solution than others. However, the compromised solution 1 has higher optimistic profit, but lower pessimistic and most likely profits than the most likely solution. The satisfaction level of compromised solution 1 is also higher than that of the most likely solution. Any model may not be formulated to exactly represent reality otherwise it may be extremely complicated. One of the reasons that the model and its solution are not accepted by the decision maker is that the model can provide only one solution but the decision maker is not satisfied with it. Therefore, it is very useful that the model can provide alternative solutions with different characteristics. The decision makers will select the one they like. The compromised model can easily be modified to provide alternative solutions. If the decision makers dislike the compromised solution 1, they may manipulate the solutions as they want. For example, they may feel that the most likely profit of $558,589 and the satisfaction level (α^m) of 0.66 are too low. They may set α^m to be 0.9 which is equivalent to the most likely profit of $616,303. In this case a constraint that $\alpha^m \geq 0.9$ should be added to the compromised model. After the model is solved, the new compromised solution called compromised solution 2 is obtained and the profits under each situation are presented in Table 5. It can be seen that the most likely profit is increased greatly while the pessimistic and optimistic profits are reduced very slightly. Values of some important decisions variables of the compromised solution 2 are presented in Table 6. Characteristics of the compromised solution 2 in Table 6 are different from those of pessimistic, most likely, and optimistic solutions.

Table 5: Profit in each situation from compromised solutions

		Real situations			α
		Pessimistic	Most-likely	Optimistic	
Solutions	Compromise1	\$301,391(α^p =0.58)	\$558,589(α^m =0.66)	\$639,486(α^o =0.58)	0.58
	Compromise2	\$293,595(α^p =0.57)	\$616,303(α^m =0.9)	\$633,748(α^o =0.57)	0.57

Table 6: Compromised solution 2 when satisfaction level $\alpha^m = 0.9$

		Period					
	Situation	1	2	3	4	5	6
Regular Demand	Pessimistic	640	800	480	960	640	480
	Most-likely	800	1,000	600	1,200	800	600
	Optimistic	960	1,200	720	1,440	960	720
Adjusted Demand	Pessimistic	640	800	560	937	817	676
	Most-likely	800	1,000	744	1,152	1,104	936
	Optimistic	960	1,200	935	1,359	1,399	1,204
Production		840	1,008	1,008	1,008	783	1,099
Overtime		0	0	0	27	7	0
Undertime		0	0	0	0	0	0
Subcontract		0	0	0	0	0	0
No. of workers		21	21	21	21	21	21
No. of hiring		11	0	0	0	0	0
No. of firing		0	0	0	0	0	0
Selling Plan		843	1,000	744	1,152	1,099	1,092
Actual Sales	Pessimistic	640	800	561	937	817	676
	Most-likely	800	1,000	744	1,152	1,099	936
	Optimistic	843	1,000	744	1,152	1,099	1,092
Promotion		-	Volume increment 20%	-	-	Discount 10%	Buy 3 units Get premium gift
Inventory	Pessimistic	300	508	955	801	1,084	1,500
	Most-likely	140	148	412	43	43	198
	Optimistic	97	105	369	0	0	0

6 Conclusions

The integrated approach to aggregate production planning and marketing promotion planning is proposed in this paper. The integrated aggregate production and marketing promotion planning model is newly developed. The integrated model is very useful for planners since it can effectively suggest optimal marketing promotion, selling, and aggregate production plans at the same time. This can avoid suboptimal solutions when these plans are performed separately. The demand and effect of promotion are uncertain input parameters to the model. They are handled by triangular fuzzy numbers representing the pessimistic, most likely, and optimistic situations. This method allows the decision makers know a possibility to get different profits under each situation. For example, the decision makers may want to maximize the most likely profit and the obtained solution is called the most likely solution. This solution yields different profits under the pessimistic, most likely, and optimistic situations. To know this information in advance, the decision makers can analyze associated risks. This paper also proposes a compromised model that attempts to simultaneously maximize the pessimistic, most likely, and optimistic profits. Since these profits are conflicting, the obtained solution is just a compromised solution. The decision makers may manipulate or control some characteristics of the compromised solution in their desirable way by adding some constraints to control the satisfaction level of pessimistic, most likely, or optimistic profit.

Further research in this area can be explained as follows. Most companies may not only be interested in the profit but also in the sales revenue or market share. They may accept to get slightly lower profit if the market share or sales revenue can be increased significantly. To handle this situation, the compromised model should be extended to consider compromised solution between the profit and revenue under pessimistic, most likely, and optimistic situations.

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