# Turbo Codes: Analysis and Design

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**Abstract**
This paper studies briefly the design and performance of turbo codes. It focuses on studying the effects of three parameters beside the termination process on the performance of turbo codes. These parameters include, number of iterations, frame size and code rate. Simulation results show that the performance proportional to these three parameters, in addition, the termination process improves the performance.

**Keywords:** Turbo codes, Design, Performance

## 1 Introduction

In 1948, Shannon issued a challenge to communications engineers by proving that the communication systems could be made arbitrarily reliable as long as a fixed percentage of the transmitted signal was redundant. He did not indicate how this could be achieved. Subsequent research has led to a number of techniques that introduce redundancy to allow for correction of errors without retransmission.

Channel coding protects digital data from errors by selectively introducing redundancies in the transmitted data. Coding involves adding extra bits to the data stream so that the decoder can reduce or correct errors at the output of the receiver. However, these extra bits have the disadvantage of increasing the data rate (bits/s) and, consequently, increasing the bandwidth of the encoded signal.

Research in coding theory has seen many proposals aimed at the construction of powerful codes using block and convolutional coding techniques [1]. Shannon theory has proved that larger block length and "random" codes possess good bit error rate (BER). However, the decoding complexity increases exponentially with the block length. On the other hand, the structure imposed on the codes in order to decrease their decoding complexity often results in relatively poor performance. As a result, approaching the channel capacity or even, more modestly, going significantly beyond the channel cutoff rate (practical limit on the highest rate at which a sequence decoder can operate) had been an unreachable dream of coding theorists for many years. In 1993 a new error correcting technique, known as turbo coding [2], was introduced and claimed to achieve near Shannon-limit error correction performance; a required $E_b/N_o$ of 0.7 dB was reported for bit error rates (BER) of $10^{-5}$ using a code rate of 1/2 in an Additive White Gaussian Noise (AWGN) channel. Many researchers have contributed to this latest technique of channel coding [3]–[12].

In this paper, we present review on the design and the performance of turbo codes including the effects of

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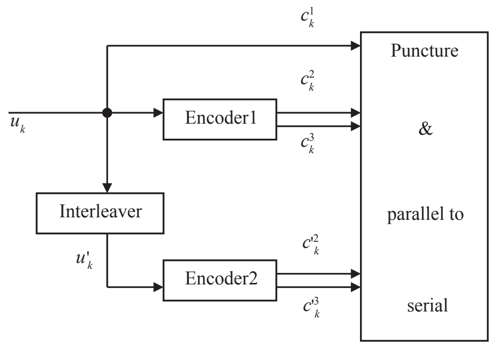



**Figure 1**: General Turbo code encoder.

several parameters such as number of iterations, frame size, and code rate.

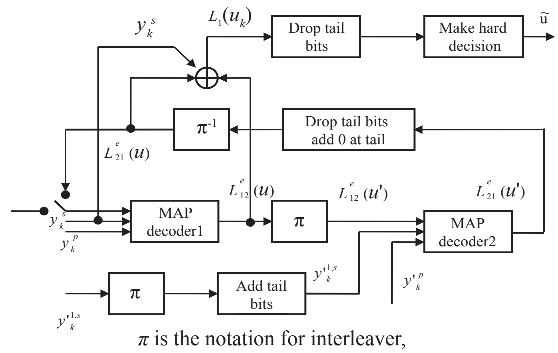
## 2 Turbo Code Encoder

A general Turbo encoder is shown in Figure 1. The Turbo code encoder employs two identical systematic recursive convolutional (RSC) encoders connected in parallel with an interleaver (the "Turbo interleaver") preceding the second recursive convolutional encoder. The two recursive convolutional encoders are called the constituent encoders of the Turbo encoder. The information bits are encoded by both RSC encoders. The first encoder operates on the input bits $\{u_k\}$ in their original order, and generate coded bits $\{c_k\}$ while the second encoder operates on the input bits $\{\grave{u}_k\}$ as permuted by the Turbo interleaver, and generates coded bits $\{\grave{c}_k\}$. If the input symbol is of length 1 and output symbol size is R, then the encoder is of code rate $r_c$=1/R. Depending on the code rate desired, the parity bits from the two constituent encoders are punctured before transmission. The role of turbo code puncturing is to periodically delete selected bits to reduce coding overhead. The tail bits will be added at the end of the transmitted frame.

## 3 Turbo Code Decoder

Figure 2 shows a block diagram of a Turbo decoder. It is constructed from two simple constituent decoders based on the Maximum a Posteriori (MAP) algorithm [13]. The turbo decoding procedure is described as follows:

A log ratio of the posteriori probability of $u_k$ conditioned on the received signal y is defined as




$\pi$ is the notation for interleaver,
$\pi^{-1}$ is the notation for de-interleaver.

**Figure 2**: Block diagram of Turbo decoder.

$$L(u_k) \triangleq \log\left[\frac{P(u_k = 1 / y_1^N)}{P(u_k = 0 / y_1^N)}\right] \tag{1}$$

The decoding decision of $\tilde{u}_k$ is made based on the sign of $L(u_k)$, i.e.,

$$\tilde{u}_k = sign\left[L(u_k)\right] \tag{2}$$

$L(u_k)$ is computed by three terms which are *L_apriori*, *L_channel*, and $L^e(u_k)$. *L_apriori* is a priori information based on the input bit $u_k$ at time $k$. It is provided by the previous decoder. *L_channel* is the received systematic bit at time $k$.

$$L(u_k) = \left[L^e(u_k) + Lc \cdot y_k^{1,s}\right] + \log\frac{\sum_{u^+} \tilde{\alpha}_{k-1}(s') \cdot \tilde{\beta}_k(s) \cdot \gamma_k^e(s',s)}{\sum_{u^-} \tilde{\alpha}_{k-1}(s') \cdot \tilde{\beta}_k(s) \cdot \gamma_k^e(s',s)}$$
$$= L\_apriori + L\_channel + L^e(u_k) \tag{3}$$

where *L_aprior* and *L_channel* denote $L^e(u_k)$ and $Lc \cdot y_k^{1,s}$ respectively. $\sum_{u^+}()$ is the summation over all the possible transition branch pair ($s_{k-1}$, $s_k$) at time $k$ given input $u_k$= 1 and $\sum_{u^-}()$ is the summation over all the possible transition branch pair ($s_{k-1}$, $s_k$) at time $k$ given input $u_k$= 0. $Lc$ is the channel reliable factor, its computation is given as the following,

$$Lc = \frac{4 \cdot A \cdot SNR\_b}{p} \tag{4}$$

where A=1 for AWGN channel, *SNR_b* is the uncoded bit-energy-to-noise-ratio ($\frac{E_b}{N_0}$), p denotes $1/r_c$, $r_c$ is code rate of the Turbo encoder.

$L^e(u_k)$ is an extrinsic information based on all parity and systematic information except the systematic value at time $k$. It can be passed on to a subsequent decoder. It is computed using the following equations:

$$L^e(u_k) \triangleq \log \frac{\sum_{u^+} \tilde{\alpha}_{k-1}(s') \cdot \gamma_k^e(s',s) \cdot \tilde{\beta}_k(s)}{\sum_{u^-} \tilde{\alpha}_{k-1}(s') \cdot \gamma_k^e(s',s) \cdot \tilde{\beta}_k(s)} \quad (5)$$

where

$$\gamma^e(s',s) = \exp\left[\sum_{i=2}^{q} \left( Lc \cdot \frac{1}{2} \cdot y_k^{i,p} \cdot c_k^i \right)\right] \quad (6)$$

$\tilde{\alpha}_k(s)$, $\tilde{\beta}_{k-1}(s')$ are the forward metric and backward metric respectively, which can be computed recursively with initial conditions described below:

$$\tilde{\alpha}_k(s) = \frac{\sum_{s'} \tilde{\alpha}_{k-1}(s') \cdot \gamma_k(s',s)}{\sum_s \sum_{s'} \tilde{\alpha}_{k-1}(s') \cdot \gamma_k(s',s)} ,$$

$$\tilde{\alpha}_0(s) = \begin{cases} 1 & \text{if } s = 1 \\ 0 & otherwise \end{cases} . \quad (7)$$

$$\tilde{\beta}_{k-1}(s') = \frac{\sum_{s} \tilde{\beta}_k(s) \cdot \gamma_k(s',s)}{\sum_s \sum_{s'} \tilde{\alpha}_{k-2}(s') \cdot \gamma_{k-1}(s',s)} ,$$

$$\tilde{\beta}_N = \begin{cases} 1 & \text{if } s = 1 \\ 0 & otherwise \end{cases} . \quad (8)$$

$$\gamma_k(s',s) \propto \exp\left[\frac{1}{2} \cdot L^e(u_k) \cdot u_k + Lc \cdot \frac{1}{2} \cdot y_k^{1,s} \cdot c_k^1\right] \exp\left[\sum_{i=2}^{q} \left( Lc \cdot \frac{1}{2} \cdot y_k^{i,p} \cdot c_k^i \right)\right] \quad (9)$$

For example, at any given iteration, decoder 1 $L_1(u_k)$ is computed as

$$L_1(u_k) = Lc \cdot y_k^{1,s} + L_{21}^e(u_k) + L_{12}^e(u_k)$$

$$\tilde{u}_k = \text{sign}[L_1(u_k)] \quad (10)$$

where $L_1(u_k)$ is given in equation 3. $L^e_{21}(u_k)$ is extrinsic information for decoder 1 derived from decoder 2, and $L^e_{12}(u_k)$ is the third term in equation 3 which is used as the extrinsic information for decoder 2 derived from decoder 1. The decoders are sharing the information with each other. The value of $L_1(u_k)$ decides the degree of the reliability of $\tilde{u}_k$.

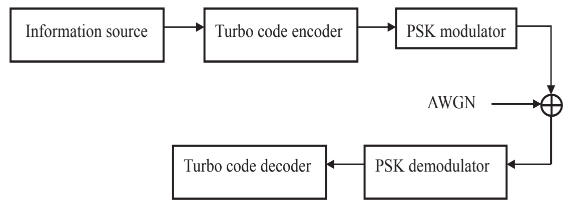



**Figure 3**: Communication system.

## 4 Simulation Results

The performance of turbo codes is conducted using computer simulation. The model of communication system used in simulation is shown in Figure 3. This model consists of information source, turbo code encoder, Phase-Shift-Keying (PSK), AWGN, PSK demodulator, and turbo code decoder. Simulation results for turbo codes are based on bit error rate (BER) over signal-to-noise-ratio ($E_b/N_o$). Three parameters are considered in studying the performance, number of iterations, frame size, and code rate, in addition to termination process.

### 4.1 *Effects of the number of iterations*

Figure 4 shows that when the number of iterations increases, the performance of the Turbo decoder improves dramatically. In other words, BER decreases dramatically. This is due to the decoder 1 and decoder 2 share the information and makes more accurate decisions. However, after the number of iterations reaches a certain value, the improvement is not significant. It can be explained that decoder 1 and 2 already have enough information, further iterations do not give them more information.

### 4.2 *Effect of the frame size (interleaver size)*

The larger the frame size, the larger the interleaver size. Therefore, it will produce larger distance by using an interleaver. When the frame size increases, the performance of the Turbo decoder improves. In other words, BER decreases. The simulation results verified this conclusion as shown in the Figure 5.

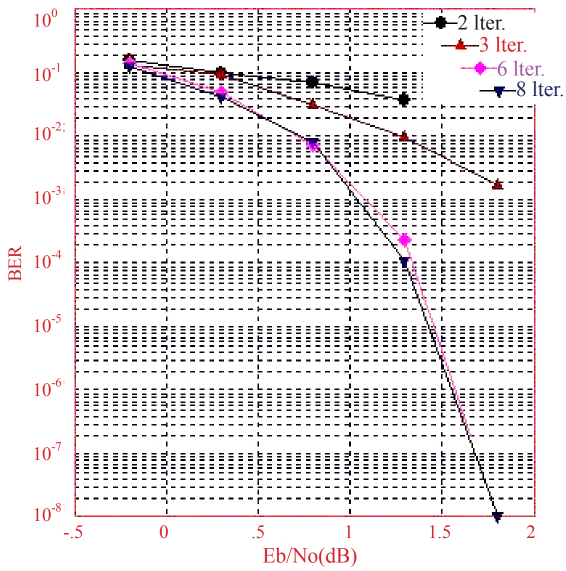



**Figure 4**: Effect of number of iterations.

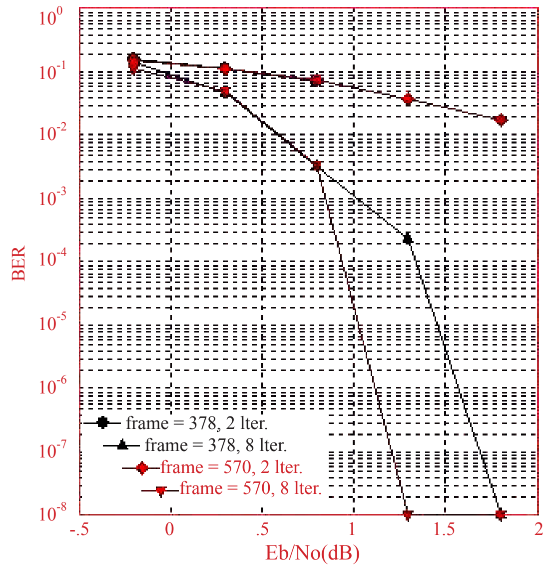



**Figure 6**: Effect of number of iterations with frame size.

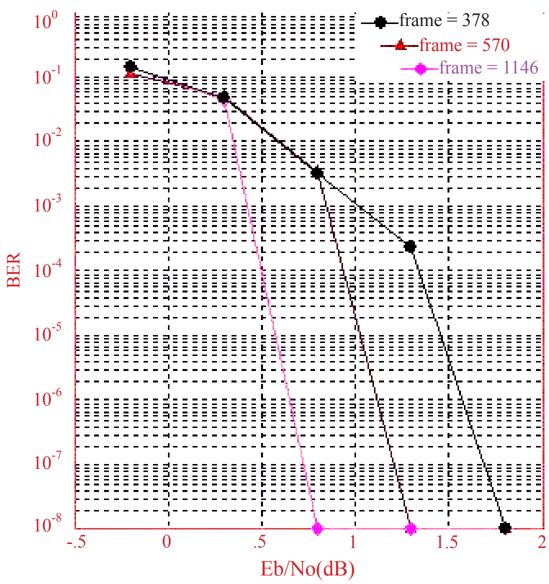



**Figure 5**: Effect of frame size.

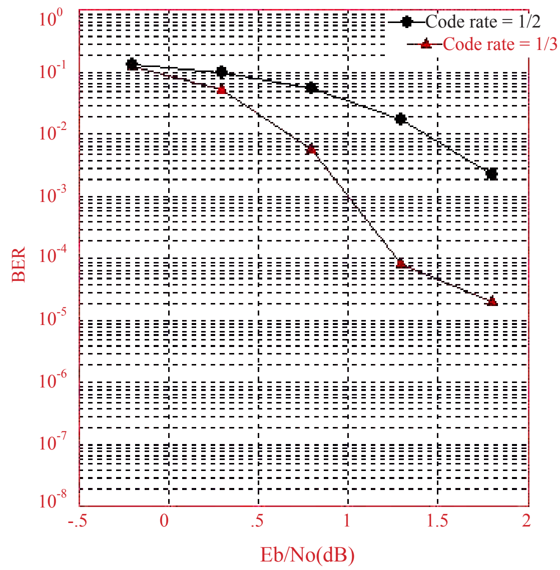



**Figure 7**: Effect of puncturing.

### 4.3 *Effect of number of iterations with the frame size*

Figure 6 shows that the performance of Turbo code does not affected by increasing the frame size when number of iterations is low. On the other hand, when number of iterations increases, the performance of the Turbo code improves dramatically.

### 4.4 *Effect of the puncturing (code rate)*

When the code rate is decreased, more bits have to be punctured. The bandwidth requirement is also decreased. This means that the performance of turbo code will also degrade in general. Figure 7 shows the effects of the puncturing on BER. The higher the code rate, the lower the BER. In the simulation,

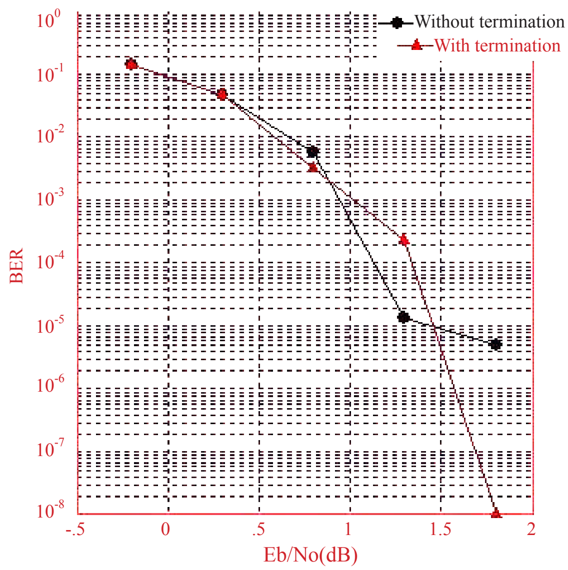



**Figure 8**: Effect of termination process.

decode iteration = 8, frame size = 378. The two curves are corresponding to code rate = 1/2 and 1/3.

### 4.5 *Effect of termination process*

Figure 8 shows the effect of termination process i.e. extra zeros added to the original data, so that Turbo code with termination has better performance than without.

## 5 Conclusions

A Turbo code system model was introduced including Turbo encoder and Turbo decoder. The performance of Turbo code system was studied under AWGN channel. The simulation results show that Turbo code is a powerful error correcting coding technique under SNR environments. It has achieved near Shannon capacity. However, there are many factors need to be considered in the Turbo code design. Firstly, a trade-off between the BER and the number of iterations need to be made, e.g., more iterations will get lower BER, but the decoding delay is longer. Secondly, the effect of the frame size on the BER. Although the Turbo code with larger frame size has better performance, the output delay is also longer. Thirdly, the code rate is another factor that needs to be considered. The higher coding rate needs more bandwidth.

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