การควบคุมตำแหน่งของระบบบอลและบีมด้วยการควบคุมแบบฟัซซี

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บทคัดย่อ

บทความนี้อธิบายวิธีการออกแบบการควบคุมแบบฟัซซีเพื่อที่จะควบคุมตำแหน่งการเคลื่อนที่ของลูกบอลของระบบบอลและบีม แบบจำลองของระบบบอลและบีมถูกใช้เพื่อหาค่าอัตราการขยายป้อนกลับให้กับตัวควบคุมอินพุตทั้งหมดสามารถถูกปรับได้ด้วยอัตราการขยายป้อนกลับนี้ ตัวควบคุมฟัซซีแบบสี่อินพุต สองอินพุต และแบบสองลูปถูกออกแบบและนำเสนอ ตัวควบคุมแบบพีไอดีถูกนำมาทดลอง ผลตอบสนองเชิงเวลาของการควบคุมทั้งหมดถูกนำมาเปรียบเทียบ ผลการทดลองพบว่าผลตอบสนองเชิงเวลาของตัวควบคุมแบบฟัซซีแบบสี่อินพุตสามารถให้ผลตอบสนองของระบบได้ดีกว่าการควบคุม แบบฟัซซีสองอินพุต แบบฟัซซีสองลูปและแบบพีไอดี

คำสำคัญ: ระบบบอลและบีม การควบคุมแบบพีไอดี การควบคุมแบบฟัซซีสองอินพุต สี่อินพุต สองลูป

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The Position Control of Ball and Beam System with Fuzzy Control

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Abstract

This paper describes a method to design a fuzzy control to control the movement of the ball and beam system. Ball and beam model is used to evaluate the feedback gains of the controller. All inputs can be adjusted by this feedback gains. The fuzzy controllers with four inputs, two inputs and two loops are designed and presented. The PID controller is implemented. The time responses of all controllers are compared. The experimental results indicate that the time responses of the fuzzy controller with four inputs give better performance than all other controllers.

Keywords: Ball and Beam System, PID Control, Fuzzy Control with Two Inputs, Four Inputs, Two Loops

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Received 8 March 2013; Accepted 26 December 2013

1. Introduction

The ball and beam system is widely known and popularly used as a laboratory model for studying the behavior of a control system. It has a simple structure but it is able to understand as a system and the control techniques for the classical and modern control design methods. The ball and beam system is a dynamic system with highly nonlinear [1]-[6]. The position of the ball rolling on the beam is changed without limit by changing the beam angle. Hence, it is an open loop unstable system. The unstable system is the most difficult problem for control and usually dangerous. Therefore, the ball and beam system is built to support the study of an unstable system. Generally, there are two types of ball and beam system models. The first model, the beam is supported in the middle and rotates against its central axis. The second model is shown in Figure1 which the beam is supported on both sides by two level arms. The ball and beam system in this paper is shown in Figure 1. The system consists of a metal ball freely rolling on the beam, DC servo motor controlling the beam, and infrared sensors detect the ball position.

The ball and beam system has been used for the result analysis of the theoretical research and new control methods. Therefore, it is standard equipment to test the classical and the modern control methods [1]-[6]. The control method for the ball and beam system have been proposed as an example in the literatures. There are many ways to control the ball and beam system such as classical, modern and combine or hybrid methods [1]-[14]. The precise mathematical model is needed in order to design the control system for a good performance and stability [2],[3],[5]. However, the nonlinear and complexity system is

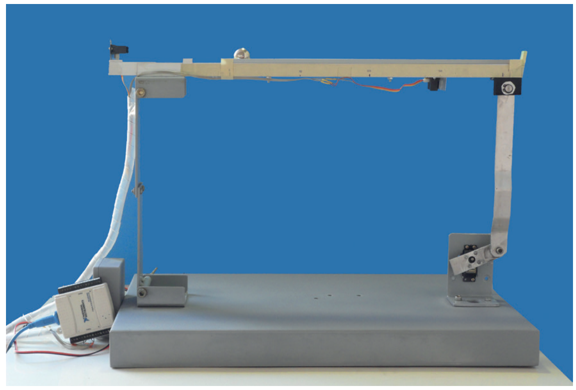


Figure 1 Ball and beam system.

difficult to know all the correct parameters. Therefore, the control methods without the mathematical model are needed to control the position of the ball on the beam system [7]-[10], [13], [14].

The object of the ball and beam system is a ball on a beam. The position of the ball on the beam is the desired position. It can’t be controlled directly but only through its acceleration which is a nonlinear system. This system can be approximated by linearization and controlled by linear control. The PD controller can be used to analyze the systems stability depending on transfer function or state space modeling [5],[6]. The parameters of the PD controller can be adjusted by the Zigler-Nichols method. It has a simple structure and provides good performance [16], [17]. The stability of ball and beam system can be analyzed by nonlinear control [2],[3],[10]-[12],[22], but these controllers are difficult and complex to implement with the real systems. This problem can be solved with the modeless controllers but depends on the experience of designers such as using fuzzy controllers which provided the satisfactory performance [7],[8],[10],[15]. Generally, the fuzzy controller for the ball and beam has one input to four inputs which increases the number of fuzzy rules.

The rule based was reduced by the sliding surface which depends on mathematical modeling [10],[12]. The number of rule bases can be reduced from 81 rules to 3 and 7 rules.

The purposes of this paper are focused on the performance comparing of difference control strategies between the PID controller and the fuzzy controller. The output behaviors of fuzzy control were studied with different inputs and independent models.

This paper presents the ball and beam control system using various fuzzy control methods. The organization of this paper is as follows: the modelling of the ball and beam system is described in section 2, the PID and Fuzzy controllers are described in section 3, the experimental setup and the results are described in section 4, and the conclusions of the results are presented in section 5.

2. Modelling of Ball and Beam System

The ball and beam system is a classic dynamic system which is highly nonlinear and open loop unstable. The aim of control is to turn the angle of beam (α) and gear (θ), in order to remain the ball at the commanded position (r). The schematic diagram for the ball and beam system is shown in Figure 2, when the angle of the beam is changed the horizontal position (unequal to zero degree), the gravity causes the ball to roll along the beam. The basic mathematical model of this system consists of the ball on the beam model and the DC servo motor [2]-[4]. Therefore, the dynamic model of the ball and beam system is analyzed. Consider the Lagrangian $L(q_i - q_i) = K - P$, and the equations of dynamic system are obtained as follows

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \frac{\partial L}{\partial q_i} = Q_i \qquad (1)$$

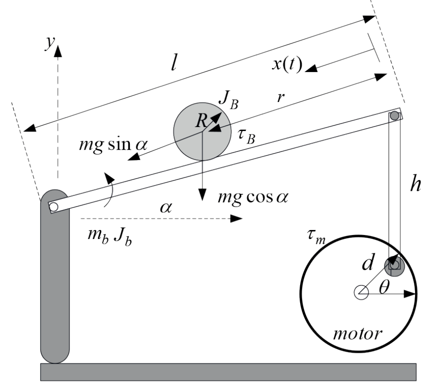


Figure 2 Schematic diagram of the Ball and Beam System.

where Q_i is the generalized force corresponding to the generalized coordinate q_i. The K and P are the kinetic and the potential energies given by equation (2), (3), [1]-[5], [7], [10].

$$K = \frac{1}{2}m_B\dot{r}^2 + \frac{1}{2}J_B(\frac{\dot{r}}{R})^2 + \frac{1}{2}(J_B + m_B r^2)\dot{\alpha}^2 + \frac{1}{2}J_b\dot{\alpha}^2$$

$$= \frac{1}{2}(m_B\dot{r}^2 + J_B(\frac{\dot{r}}{R})^2 + (J_B + m_B r^2)\dot{\alpha}^2 + J_b\dot{\alpha}^2) \qquad (2)$$

$$P = \frac{l}{2}m_b g \sin\alpha + m_B g r \sin\alpha \qquad (3)$$

Then, the Lagrange's equation is given by

$$L = \frac{1}{2}(m_B\dot{r}^2 + J_B(\frac{\dot{r}}{R})^2 + (J_B + m_B r^2)\dot{\alpha}^2 + J_b\dot{\alpha}^2)$$

$$-\frac{l}{2}m_b g \sin\alpha - m_B g r \sin\alpha \qquad (4)$$

where m_B is the mass of the ball, r is the position of the ball, $\dot{r}$ is the velocity of the ball, g is the gravitational constant, J_B is the moment of inertia of the ball, J_b is the moment of inertia of the beam, R is radius of the ball, α is the angle of the beam which is approximated as linear by $\alpha = (d / l)\theta$, $\dot{\alpha}$ is the angular velocity of the beam, l is the length of the bean and m_b is the mass of the beam. The value of J_B is given by $J_B = 2mR^2 / 5\ kg - m^2$ for the solid ball and the value of J_b is given by $J_b = ml^2 / 3\ kg - m^2$ for the slender rod at the end. From the equation (1), the dynamic equations of motion for ball and beam system are obtained as follow:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}}\right] - \frac{\partial L}{\partial x} = 0 \quad (5)$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\alpha}}\right] - \frac{\partial L}{\partial \alpha} = \tau \quad (6)$$

where τ is the generalized force. From equation (4) substitute in equations (5) and (6), therefore, the equation of motion of the ball and beam system is given by

$$(m_B + \frac{J_B}{R^2})\ddot{r} - m_B\dot{\alpha}^2 r + m_B g \sin\alpha = 0 \quad (7)$$

$$(J_B + J_b + m_B r^2)\ddot{\alpha} + 2m_B r\dot{r}\dot{\alpha} + (\frac{l}{2}m_b + m_B r)g\cos\alpha = \tau \quad (8)$$

where

$$\frac{\partial L}{\partial \dot{r}} = (m_B + \frac{J_B}{R^2});\ \frac{\partial L}{\partial r} = m_B r\dot{\alpha}^2 - m_B g\sin\alpha$$

$$\frac{\partial L}{\partial \dot{\alpha}} = (J_B + J_b + m_B r^2)\dot{\alpha};\ \frac{\partial L}{\partial \alpha} = -(\frac{l}{2}m_b + m_B r)g\cos\alpha = \tau$$

The state space modeling of the ball and beam system is given as follows: Let $r = x$, $\dot{r} = x_2$, $\alpha = x_3$ and $\dot{\alpha} = x_4$.

$$\dot{x}(t) = [\dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_4]^T \quad (9)$$

Thus

$$\dot{x}(t) = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = m_B a(x_1 x_4^2 - g\sin x_3) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = b(\tau - 2m_B x_1 x_2 x_4 - (\frac{l}{2}m_b + m_B x_1)g\cos x_3) \end{cases} \quad (10)$$

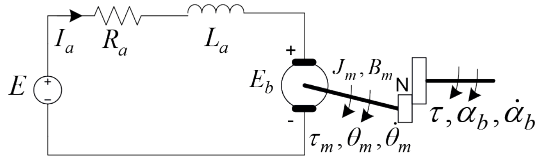


Figure 3 Equivalent circuit of the DC servomotor.

$y = x_1$

where $a = \dfrac{1}{(m_B + J_B\dfrac{1}{R^2})}$; $b = \dfrac{1}{(J_b + J_B + m_B x_1^2)}$

The DC servomotor is used to drive the beam shown in Figure 3. The equations for dynamic model of the DC motor are given as [2].

$$E = L_a\frac{dI_a}{dt} + R_a I_a + E_b \quad (11)$$

$$\tau_m = K_t I_a$$

$$= J_m\frac{d\dot{\theta}_m}{dt} + B_m\dot{\theta}_m \quad (12)$$

$$E_b = k_b\dot{\theta}_m$$

where E is voltage of the DC servomotor, L_a and I_a are the armature inductance and current respectively, R_a is the armature resistance, E_b and θ_m are the back EMF voltage and the angular speed respectively. Thus, the following energy equivalent holds [2]

$$E_b.I_a = \tau_m.\dot{\theta}_m \quad (13)$$

Substituting equation (12) into equation (13), then the motor constant is given by

$$k_e = k_t$$

Since L_a is small enough to be neglected, (11) and (12) can be simplified as

$$E = R_a I_a + K_b \dot{\theta}_m \quad (14)$$

where

$$I_a = \frac{\tau_m}{K_t}$$

The gear ratio of the motor is replaced as N which can be shown in Figure 3 and the relationship is given as follows:

$$\frac{\theta_m}{\alpha_b} = \frac{\dot{\theta}_m}{\dot{\alpha}_b} = \frac{\tau}{\tau_m} = N \quad (15)$$

Substituting equation (14) into equation (12), then the torque of the ball and beam system can be shown in equation (16).

$$\tau = \frac{NEK_t}{R_a} - \frac{N^2 K_t^2 \dot{\alpha}_b}{R_a} \quad (16)$$

Thus, the state space of torque can be derived as

$$\tau = \frac{NK_t u}{R_a} - \frac{N^2 K_t^2 x_4}{R_a} \quad (17)$$

where

$$E = \frac{R_a \tau}{K_t N} + K_t N \dot{\alpha}_b$$

$$\frac{\tau}{R_a K_t N} = E - K_t N \dot{\alpha}_b$$

$$\frac{\tau}{N} = \frac{EK_t}{R_a} - \frac{NK_t^2 \dot{\alpha}_b}{R_a}$$

Substituting equation (17) into equation (10), the state space equation of the ball and beam system and DC motor can be derived as

$$\dot{x}_1 = x_2 \quad (18)$$

$$\dot{x}_2 = a x_1 x_4^2 - a g \sin x_3 \quad (19)$$

$$\dot{x}_3 = x_4 \quad (20)$$

$$\dot{x}_4 = \frac{bNK_t u}{R_a} - \frac{bN^2 K_t^2 x_4}{R_a} - 2b m_B x_1 x_2 x_4 - b(\frac{l}{2} m_b + m_B x_1) g \cos x_3 \quad (21)$$

Because of the angle changing is small, therefore the results can be defined as $\sin x \approx x$ and $\cos x \approx 1$. Rewriting equations (18) to (21) in state-space of the system, we get

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{m_B g}{(m_B + J_B \frac{1}{R^2})} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m_B g}{J_b + J_B} & 0 & 0 & -\frac{N^2 K_t^2}{Ra(J_b + J_B)} \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \\ \alpha \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{NK_t}{R_a(J_b + J_B)} \end{bmatrix} u$$

$$y = x_1 \quad (22)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -7.007 & 0 \\ 0 & 0 & 0 & 1 \\ -36.323 & 0 & 0 & -19.216 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \\ \alpha \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 37.709 \end{bmatrix} u \quad (23)$$

All inputs can be adjusted by using gains g_1, g_2, g_3 and g_4. The gains can be found by using Linear Quadratic Regulator (LQR) design which is based on linearized model. From equation (23), the feedback gains K can be found by the LQR techniques.

$$K = [-2.3517 \quad -1.7957 \quad 5.6130 \quad 0.7398] \quad (24)$$

Table 1 Physical parameters of the ball and beam system

Symbol	Definition	Value
m_B	Mass of the ball	0.05 kg
m_b	Mass of the beam	0.150 kg
l	Beam length	0.52 m
R	Radius of the ball	0.0125 m
J_B	Ball inertial	3.125 x 10^{-6} kg-m^2
J_b	Beam inertial	0.0135 kg-m^2
d	Lever length	0.04 m
R_a	Armature resistance	4.46 Ω
g	Gravitational force	9.81 m/s^2
N	Gear ratio	280
K_t	Torque constant	0.00182 Nm/A
n	Speed	0.14 sec/60 degree

3. Control Systems

This section describes the difference control strategies between PID controller and fuzzy controller. The fuzzy controller can be separated into three control methods that are the fuzzy with four inputs, the fuzzy with two inputs, and the fuzzy with two loops.

3.1 PID Control

The Proportional-Integral-Derivative (PID) algorithm is the most popular control algorithm used in industries and general automation systems. The prominent point of this is relatively easy to find the control parameters by adjusting the parameter values between the real control experiment or use the Ziegler-Nichols method. The output equation of PID is shown in the equation (25) [18], [20] and the block diagram is shown in Figure 5,

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \qquad (25)$$

where $e(t)$ is an error signal by comparing the reference input $r(t)$ to the feedback variable $y(t)$, τ is variable of integration , K_p is the proportional gain, K_i is integral gain and K_d is derivative gain. The Ziegler-Nichol methods for determining the parameters of a PID controller are shown in Table 2. The K_u and P_o are ultimate gain and oscillation period respectively.

Table 2 Tuning formula of the Zigler-Nichols method [20]

Controller	K_p	K_i	K_d
P	0.5 K_u	-	-
PI	0.45 K_u	1.2 K_p/P_o	-
PID	0.6 K_u	2 K_p/P_o	$K_pP_o/8$

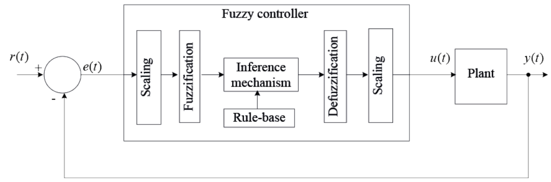


Figure 4 Block diagram of fuzzy controller.

3.2 Fuzzy Control

The fuzzy control is a control system based on a mathematical system of fuzzy logic which is widely used in automation and general control system. It was first proposed by Professor Lotfi A. Zadeh in 1965 from the University of California at Berkeley. A block diagram of a fuzzy control system is shown in Figure 4. The fuzzy controller is composed of the following four elements [10], [15]:

1. Fuzzification: converts input variables into linguistic values of the fuzzy set.

2. Inference mechanism: emulates the expert's decision in an interpreting which associated with membership function, logical and IF-THEN rules.

3. Rule-base: a set of If-Then rules which contains expert knowledge to control the system.

4. Defuzzification: converts the conclusions of the inference mechanism into actual inputs for the process or for the plant. Center of Gravity (COG) defuzzification method or called Center of Area is used in this paper and is given by

$$COG : y_q^{crisp} = \frac{\sum_{i=1}^{R} c_q^i \int_{y_p} \mu_{A_q^i}(y_q) dy_q}{\sum_{i=1}^{R} \int_{y_p} \mu_{A_q^i}(y_q) dy_q} \qquad (26)$$

where R is the total number of control rules, c_q^i denotes the center value of y_q which is the output region of rule i, and y_q is the current degree of membership function $\mu_{A_q^i}$ for sensory reading input A for the i rule.

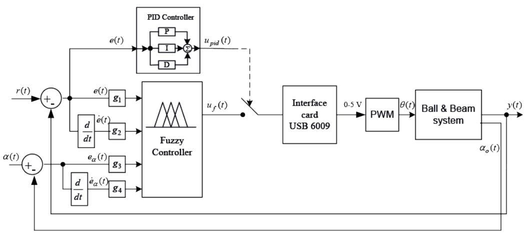


Figure 5 Block diagram of PID and fuzzy controller with four inputs for ball and beam system.

The block diagram of the control system for the ball and beam system is shown in Figure 5 which includes the PID and the fuzzy controllers. Both controllers work independently. Figure 5, the input of fuzzy controller consists of the position error of the ball (e), the change of the position error of the ball ($\dot{e}$), the angle error of the beam (e_α) and the changing of the beam error of the angle ($\dot{e}_\alpha$). The input equations are shown in equations (27) to (30).

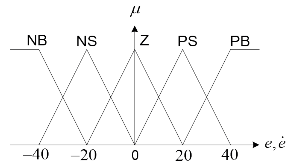
$$e(k) = r(k) - y(k) \tag{27}$$

$$\dot{e}(k) = e(k) - e(k-1) \tag{28}$$

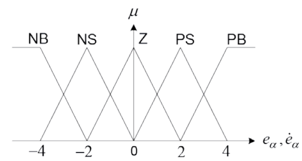
$$e_\alpha(k) = \alpha(k) - \alpha_o(k) \tag{29}$$

$$\dot{e}_\alpha(k) = \alpha(k) - \alpha(k-1) \tag{30}$$

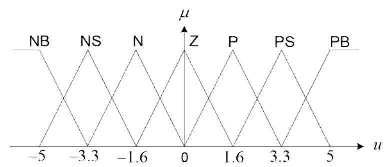
where $r(k)$ is reference position of the ball, $y(k)$ is output of the ball position , $\alpha(k)$ is reference angle of the beam and $\alpha_o(k)$ is output of the beam angle. The gains g_1, g_2, g_3 and g_4 are the feedback gain to adjust the actual inputs of the fuzzy controller which is derived from the equation (24). The triangular shape is used for the membership functions of the fuzzy controller. The input membership of the ball position error and derivative of ball position error are defined in an



a) The input membership of ball position error and derivative of ball position error.



b) The input membership of beam angle error and derivative of beam angle error.



c) The output membership

Figure 6 The membership function for fuzzy controller with four inputs.

interval from [-40, 40] that is the length of the beam 40 centimetres as shown in Figure 6 a). The beam angle error membership and derivative of the beam angle error membership are defined in an interval from [-4, 4] that is the range of the beam angle ± 4 degrees as shown in Figure 6 b). The range of output membership is defined in an interval from [-5, 5] that is the angle of the beam ± 5 degrees. The linguistic values of input and output memberships can be explained as follows: N is Negative, Z is Zero, P is Positive, NB is Negative Big, NS is Negative Small, PS is Positive Small and PB is Positive Big.

The membership function of fuzzy controller with four inputs consist of inputs and output. The input membership functions are defined by error and derivative of error which derived from the feedback signals of the system and are compared to the reference signals. Therefore, there are position error, derivative of position error, angle error and derivative of angle error. The triangular shape is used for the input membership functions and the memberships are defined a symmetric in order to provide the linearity of the output. The membership of input functions are defined as five memberships to provide the accurate input. The range of input function is defined by the error value of the ball position and the error value of the beam angle which are defined in the range of ± 40 cm and ± 4 degree, respectively. The reason defined the range of ± 40 cm and ± 4 degrees because the dynamic of the ball and beam system have the high dynamic and the acceleration. Therefore, if define the range near the equilibrium such as ± 1, it will affect the oscillation of the output signal. The output membership function is defined by the coordinate of the beam angle which are defined the beam angle of ± 5 degrees. The membership of output function is defined by seven memberships for resolution of the fuzzy output to control the motor angle of ± 45 degrees.

For the fuzzy rule base building is the mapping of the inputs to the outputs by using a set of the IF-THEN rules. The inputs and outputs of the fuzzy system are associated with the premise and consequent respectively. The multi-input single-output forms of linguistic rule for this fuzzy system are shown as follows:

R^1: IF e is NB AND $\dot{e}$ is NB AND e_α is NB AND $\dot{e}_\alpha$ is NB THEN u is NB

. .
R^{313}: IF e is Z AND $\dot{e}$ is Z AND e_α is Z AND $\dot{e}_\alpha$ is Z THEN u is Z

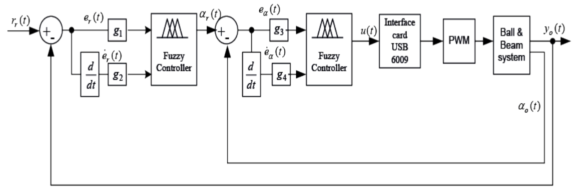


Figure 7 Block diagram of the two loops fuzzy controller for the ball and beam system.

. . .
R^{625}: IF e is PB AND $\dot{e}$ is PB AND e_α is PB AND $\dot{e}_\alpha$ is PB THEN u is PB

The total numbers of rule base are m^n where m is the membership number and n is the input variable number. In this case, there are five memberships for all four inputs; therefore, the rule base of fuzzy controller with four inputs are 625 possible rules which are shown in Table 3.

The fuzzy control system with two loops is shown in Figure 7, which is composed of the ball position or outer loop and the beam angle or inner loop. The outer loop controls the position of the ball on the beam by manipulating the servo motor angle and the inner loop controls the position of servo angle to adjust the beam angle. The input equations of inner loop are shown in equations (31) to (32).

$$e_\alpha(k) = \alpha_r(k) - \alpha_o(k) \quad (31)$$

$$\dot{e}_\alpha(k) = e_\alpha(k) - e_\alpha(k-1) \quad (32)$$

The input equations of outer loop are shown in equations (33) to (34).

$$e_r(k) = r_{ref}(k) - y_o(k) \quad (33)$$

$$\dot{e}_r(k) = e_r(k) - e_r(k-1) \quad (34)$$

Table 3 The rule base table for the four inputs fuzzy controller

u		$e_\alpha, \dot{e}_\alpha$																								
		NB, NB	NB, NS	NB, Z	NB, PS	NB, PB	NS, NB	NS, NS	NS, Z	NS, PS	NS, PB	Z, NB	Z, NS	Z, Z	Z, PS	Z, PB	PS, NB	PS, NS	PS, Z	PS, PS	PS, PB	PB, NB	PB. NS	PB, Z	PB, PS	PB, PB
$e, \dot{e}$	NB, NB	NB	NB	NB	NB	NB	NB	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z
	NB, NS	NB	NB	NB	NB	NB	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P
	NB, Z	NB	NB	NB	NB	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P
	NB, PS	NB	NB	NB	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P
	NB, PB	NB	NB	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P
	NS, NB	NB	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P
	NS, NS	NB	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P
	NS, Z	NB	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P
	NS, PS	NS	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P
	NS, PB	NS	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS
	Z, NB	NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS
	Z, NS	NS	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS
	Z, Z	NS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS
	Z, PS	NS	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS
	Z, PB	NS	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS
	PS, NB	NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS
	PS, NS	N	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS
	PS, Z	N	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB
	PS, PS	N	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB
	PS, PB	N	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB	PB
	PB, NB	N	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB	PB	PB
	PB, NS	N	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB	PB	PB	PB
	PB, Z	N	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB	PB	PB	PB	PB
	PB, PS	N	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB	PB	PB	PB	PB	PB
	PB, PB	Z	P	P	P	P	P	P	P	P	PS	PS	PS	PS	PS	PS	PS	PS	PB	PB	PB	PB	PB	PB	PB	PB

where $r_r(k)$ is reference the input signal, $y_o(k)$ is the output signal of the ball position, $e_r(k)$ is the position error of the ball, $\dot{e}_r(k)$ is the change of the position error of the ball, $\alpha_r(k)$ is the reference input signal derived from the defuzzification of the outer loop compare to angle output of the beam, $\alpha_o(k)$ is the output of the beam angle, $e_\alpha(k)$ is the position error of the beam angle, $\dot{e}_\alpha(k)$ is the change of the position error of the angle beam. The gain g_1 to g_4 are the feedback gains.

The membership function of the fuzzy controller with two loops is defined by using the same principle as the fuzzy controller with four inputs. For the fuzzy controller with two loops must be defined as the output of the outer loop relative to the feedback signal of the inner loop. In this section, the output membership function of the outer loop is defined in the range of ± 4 degrees as shown in Figure 9 because the feedback signal of the beam angle in the range of ± 5 degrees and the output membership function of the inner loop is defined the same as the fuzzy controller with four inputs as shown in Figure11.

The input membership function for the outer loop is shown in Figure 8. The universe of discourse for the outer loop is defined in an interval [-40,40]. The output membership of the outer loop is shown in Figure 9 and the universe of discourse is defined in an interval [-4,-4] that is the range of the beam angle ± 4 degrees. The input membership function for the inner loop is shown in Figure 10 which is defined in an interval [-4,-4]. The membership function for the output of the outer loop and the input of inner loop is defined in the corresponding range. The output membership

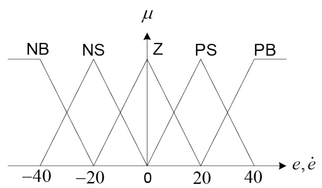


Figure 8 The input membership function for outer loop.

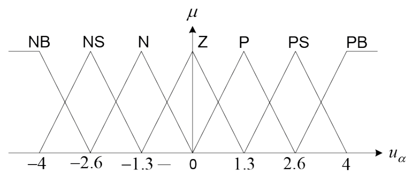


Figure 9 The output membership function for outer loop.

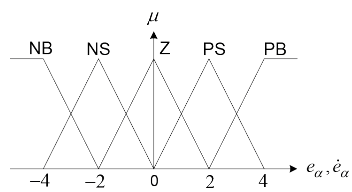


Figure 10 The input membership functions for inner loop.

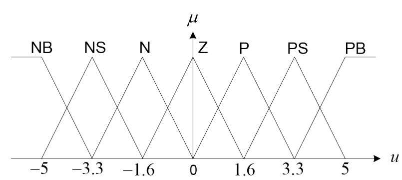


Figure 11 The output membership functions for inner loop.

function of the inner loop is defined interval [-5,-5] that is the angle of the beam ± 5 degrees as shown in Figure11. The rule bases are shown in Table 4.

Table 4 The rule base table for the two inputs and two loops fuzzy controller

u_α		$\dot{e}_r$					u		$\dot{e}_\alpha$				
		NB	NS	Z	PS	PB			NB	NS	Z	PS	PB
e_r	NB	NB	NB	NB	N	Z	e_α	NB	NB	NB	NB	N	Z
	NS	NB	NS	NS	Z	PS		NS	NB	NS	NS	Z	PS
	Z	NS	N	N	PS	PS		Z	NS	N	N	PS	PS
	PS	N	Z	Z	PS	PB		PS	N	Z	Z	PS	PB
	PB	Z	P	PS	PB	PB		PB	Z	P	PS	PB	PB

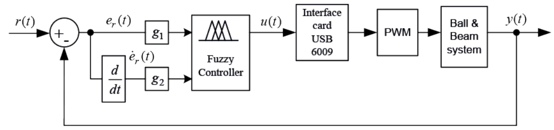


Figure 12 Block diagram of two inputs fuzzy controller for ball andbeam system.

The rule bases of this controller are shown in Table 4 which can be described the linguistic rule as follow:

R1: IF e_r is NB AND $\dot{e}_r$ is NB THEN u_α is NB, i = 1,2,..25 and

R1: IF e_α is NB AND $\dot{e}_\alpha$ is NB THEN u is NB, i = 1,2,..25

The control system for the fuzzy controller with two inputs is shown in Figure 12 which is considered only the position of the ball. The membership functions in Figure 8 and Figure 11 are used for input and output membership functions of the system. The rule base in Table 4, the first part is used for this controller.

4. Experimental Setup and Results

The experiments were setup which consists of the ball and beam system (see the parameters in Table 1), NI USB6009, PWM set, DC servomotor, and LabVIEW program. The infrared sensors are used for the ball position on the beam and the beam angle. The voltage 5 V is used to supply the DC servomotor. USB interface (NI USB6009) is used to link between output of LabVIEW and input of the PWM which is the analog signal 0-5 V [19], shown in Figure 13.

The PWM is used to control the angle of the DC servomotor from -45 to 45 radians or about 4 degrees of the beam angle. The aim of the experiment is the stability of the ball position moving on the beam which

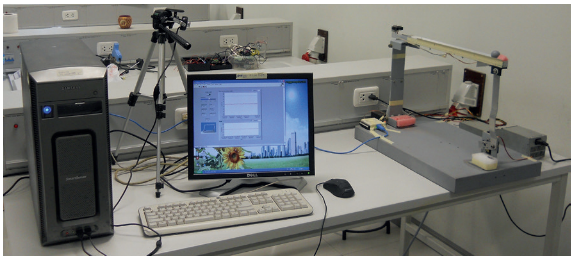


Figure 13 Experimental setup of the ball and beam system.

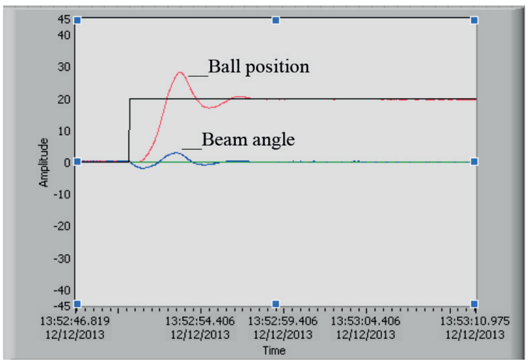


Figure 14 The ball position and beam angle response using the PID controller.

is defined from 1 to 40 cm. The beam angle is defined from -4 to 4 degree. The gain g_1, g_2, g_3, and g_4 are defined by equation (24). The moving ball is associated with the beam angle and the behavior control of the ball moving on the beam. The ball position and the beam angle will be sent back as the feedback signal to compare the reference commands. If the error is zero, the ball will be stopped and hold the current position. The experiment of the PID, fuzzy (two inputs), fuzzy (four inputs) and Fuzzy (two loops) controllers is presented. The performance of the control systems are compared by the time responses. The PID and the fuzzy toolkits for LabVIEW program were installed. The parameters of the PID controller were calculated by the Ziegler-Nichols method which is shown in

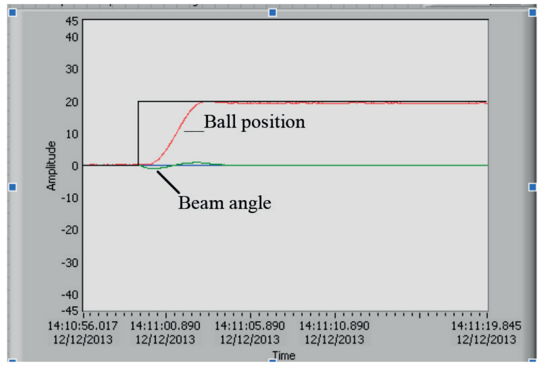


Figure 15 The ball position and beam angle response using the two inputs fuzzy controller.

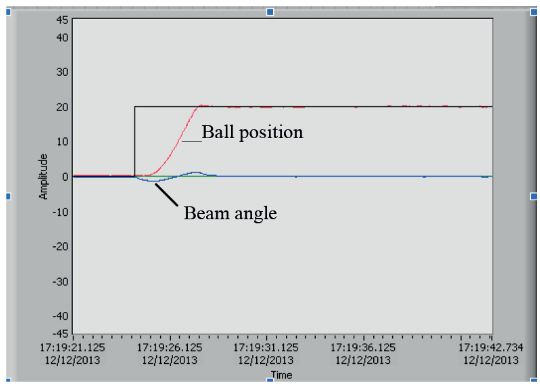


Figure 16 The ball position and beam angle response using the four inputs fuzzy controller.

Table 2. Thus, the K_p, K_i and K_d parameters are 0.024, 0.011 and 0.013 respectively. The time response of the PID controller is shown in Figure 14 and concluded as shown in Table 5. The time responses of fuzzy controller are shown in Figure15 to Figure 17. The total time response of all controllers is shown in Figure18.

The rise time (T_r), the delay time (T_d), the maximum time (T_m), the settling time (T_s), and the steady state error (e_{ss}) are used to analyze the time response of the control systems and shown in Table 5 [21]. The experiment result shown that the PID controller has more overshoot and the response gave the smallest rise

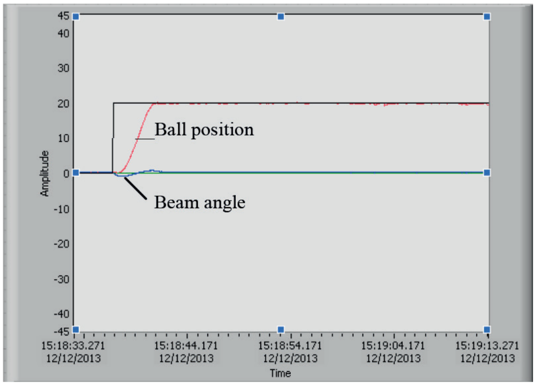


Figure 17 The ball position and beam angle response using the two loops fuzzy controller.

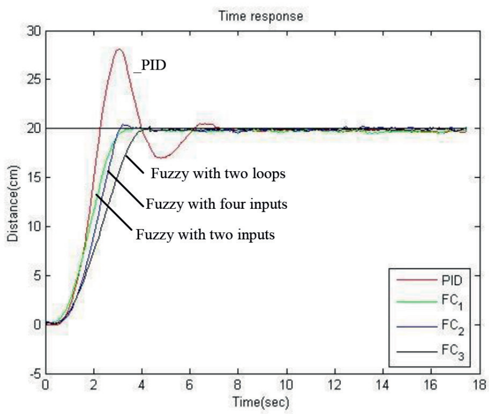


Figure 18 The ball position response for the four controllers.

time but gave the most settling time. The fuzzy with two loops gave the most rise time but gave the smallest settling time. The steady state error of the fuzzy control with four inputs is less than other controllers.

Table 5 Time response of the ball position

Controller	T_r (sec)	T_d (sec)	T_m (sec)	T_s(2%) (sec)	e_{ss} (%)
PID	1.16	1.76	3.08	6.90	1.10
Fuzzy (two inputs)	1.80	1.86	3.28	3.52	1.38
Fuzzy (four inputs)	1.76	2.10	3.22	3.32	0.09
Fuzzy (two loops)	2.30	2.34	4.21	4.34	0.55

5. Conclusions

This paper presents the ball and beam system by applying the fuzzy controllers. The two inputs, four inputs and two loops of the fuzzy controller are implemented and compared with the PID controller. The ball position and the beam angle were considered as the inputs of the fuzzy controller. The time responses of the ball and beam system with three fuzzy controllers shown a slight difference. The controller design for two inputs is simple but time response resemble the four inputs. The four inputs may be difficult for setting the rules and uses a lot of memory for source code. The rules can be reduced by the two loops from 625 rules to 50 rules. The controller design for two loops is more difficult than two inputs because the outer loop and the inner loop need to set the values accordingly. However, the fuzzy controller with four inputs can provide better control performance because it has more input variables and rules than all other controllers in this paper. But if there are too many rules, it will affect the time of processing. Therefore, it may be inappropriate for systems with high dynamics.

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