Research Article

Average Run Length of Double Modified Exponentially Weighted Moving Average Control Chart by Numerical Integral Equation

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Abstract

Through the use of Numerical Integral Equation (NIE) techniques—specifically, the Gaussian, Midpoint, Trapezoidal, and Simpson's rules—and the Double Modified Exponentially Weighted Moving Average (DMEWMA) control chart, this study aims to investigate the Average Run Length (ARL) approximation. Assuming that process data follow continuous probability distributions, specifically the exponential and Weibull distributions, the analytical framework is developed. Additionally, a thorough performance comparison is conducted between the DMEWMA control chart and two well-known substitutes: the Exponentially Weighted Moving Average (EWMA) control chart and the Modified Exponentially Weighted Moving Average (MEWMA) control chart. This comparative analysis is based on two critical performance indicators: the out-of-control Average Run Length (ARL_1) and computational efficiency, as quantified by CPU processing time. The empirical results demonstrate that all NIE-based methods produce ARL estimates that are statistically indistinguishable, affirming their mutual accuracy. Nonetheless, with respect to computational performance, the Midpoint and Trapezoidal rules exhibit superior efficiency, achieving reduced processing times. Furthermore, at all levels of shift magnitude, the DMEWMA control chart continuously outperforms the MEWMA and EWMA charts in terms of sensitivity to changes in the process mean. All things considered, the results of this study highlight the effectiveness and usefulness of using NIE techniques for ARL estimation in sophisticated statistical process control. The proposed methodology not only yields precise and computationally efficient results but also exhibits strong applicability to a wide array of real-world datasets, thereby reinforcing its potential as a robust and versatile tool for contemporary process monitoring.

Keywords: Double modified exponentially weighted moving average control chart, Gaussian rule, Midpoint rule, Simpson's rule, Trapezoidal rule

1 Introduction

Variations are inevitable in industrial production processes and may arise from various factors, potentially affecting the quality of the final product. To ensure effective quality control, the concept of product quality control has been developed to maintain the process within defined standards. It serves to alert manufacturers when deviations from specified criteria occur, thereby preventing defects and minimizing production-related issues.

Statistical Process Control (SPC) refers to the systematic application of statistical methods to monitor, control, and improve the quality and consistency of a production process, thereby ensuring its efficient and stable operation. In 1976, Dr. Kaoru Ishikawa's seminal work, Guide to Quality Control [1], presented a whole suite of methods for quality improvement. The seven tools—the check sheet, histogram, cause-and-effect diagram, Pareto chart, scatter diagram, stratification, and control chart—are now commonly known as the seven quality control (7-QC) tools. Of these, the Control Chart continues to be one of the most popular and useful instruments in SPC for identifying process variance and upholding quality standards.

Control charts are a core tool in Statistical Process Control (SPC), designed in various forms to suit the specific nature of production processes and the variables being monitored. Detailed explanations on the development and application of each type will be provided in the following sections.

The control chart was initially created by Shewhart [2], who also laid the groundwork for SPC. Systematic quality control in industrial manufacturing began with his efforts. Consequently, it is referred to as the Shewhart control chart. In contrast to other control chart styles, the Shewhart chart detects process irregularities more slowly. As a result, memory-type control charts were recommended. In 1959, Roberts [3] developed the Exponentially Weighted Moving Average (EWMA) control chart, which gives importance to both historical and current data. This type of chart, known as a memory control chart, assigns weights to past and present data to improve process monitoring. To enhance the capacity to detect minute alterations in the procedure, Patel and Divecha [4] and Khan *et al*. [5] created the Modified Exponentially Weighted Moving Average (MEWMA) control chart. Although it can recognize both little and large movements, the MEWMA chart is more effective at detecting minor shifts. Alevizakos *et al*. [6] proposed the Double Modified Exponentially Weighted Moving Average (DMEWMA) control chart. This control chart incorporates the concept of smoothing into the MEWMA chart, resulting in improved ability to detect small shifts in the process. There are several types of continuous probability distributions that can be used to model lifetime data. This study focuses on the exponential and Weibull distributions, as they are well-suited for skewed data and are effective in modeling the time between events.

The exponential distribution is commonly used to simulate the time interval between randomly occurring events at a constant rate, such as the failure time of electronic devices or the death rate of cancer patients. The "memoryless property," which asserts that the probability of an event occurring in the future is independent of the amount of time that has elapsed, is a crucial aspect of this distribution. For modeling skewed data, the Weibull distribution is incredibly adaptable. It is frequently employed in systems or product reliability analysis. This distribution can represent hazard functions that increase, decrease, or remain constant over time, which helps describe the product's lifetime at different stages.

The Average Run Length (ARL) is typically used to assess the effectiveness of control charts. When the process is under control, it can be categorized as the ARL_0; otherwise, it can be classified as the ARL_1. The ARL can be estimated using a variety of methods, including explicit formulas, the Numerical Integral Equation (NIE) method, the Markov Chain Approach (MCA), the Martingale Approach (MA), and Monte Carlo simulation (MC). Champ and Rigdon [7] compared the methods for estimating the Average Run Length (ARL) of CUSUM and EWMA control charts using the Numerical Integral Equation (NIE) approach and the Markov Chain methodology. Areepong and Sukparungsee [8] used the NIE approach to calculate the ARL of the EWMA control chart when the data had a lognormal distribution. Their study demonstrated that the NIE method, which calculates the ARL using the Gauss-Legendre Quadrature rule, produced accurate results and required less processing time than the Monte Carlo Simulation (MC) methodology.

The ARL results from the explicit formulas on the MEWMA control chart for a first-order autoregressive process (AR(1)) in the presence of exponential white noise were contrasted with those from the NIE technique by Phanthuna *et al*. [9] (see more [10]). The results of the investigation showed that there was very little difference between the two approaches' performances. For determining the ARL, however, the NIE approach worked better. A NIE method was provided by Neammai *et al*. [11] to estimate the ARL of the DMEWMA control chart for moving average process order q (MA(q)) in order to check the least time consuming. The ARL of the CUSUM control chart for a long-memory fractionally integrated autoregressive process with exponential white noise was estimated by Bualuang and Peerajit [12] using the NIE approach. According to the study, the NIE approach yields precise answers with a relative percentage change of less than 0.25% that are in line with the analytical calculations.

Numerical Integral Equation (NIE) approaches have been the subject of numerous investigations. Nevertheless, there hasn't been much research done on the ARL estimate employing NIE techniques for DMEWMA control charts. Therefore, the goal of this article is to examine different NIE techniques used for the DMEWMA control chart when the data follow Weibull and exponential distributions, such as the

Midpoint Rule, Gaussian Rule, Trapezoidal Rule, and Simpson's Rule. Additionally, using metrics like Average Run Length (ARL) and Average Extra Quadratic Loss (AEQL), the DMEWMA control chart's performance is contrasted with that of the MEWMA and EWMA control charts. Lastly, the control charts under study are used with actual data.

2 Methodology

2.1 *Control chart*

2.1.1 Exponentially Weighted Moving Average Control Chart (EWMA)

Proposed by Roberts [2], the test statistic of the EWMA control chart is given by Equation (1) as follows:

$$Z_t = (1-\lambda_1)Z_{t-1} + \lambda_1 X_t \text{ ; } t = 1,2,... \quad (1)$$

where λ_1 is a smoothing parameter of the EWMA control chart ($0 < \lambda_1 \le 1$), Z_t is the EWMA statistic and X_t is an observation from a process. The asymptotic control limits of the control chart are given by Equation (2),

$$UCL / LCL = \mu_0 \pm L_1 \sigma \sqrt{\frac{\lambda_1}{2-\lambda_1}} \quad (2)$$

where μ_0 and σ are the expectation and the standard deviation when the process is in control, respectively, and L_1 is the control coefficient of the EWMA control chart.

2.1.2 Modified Exponentially Weighted Moving Average Control Chart (MEWMA)

The MEWMA control chart is an expansion of the EWMA control chart, which was proposed by Patel and Divecha [3]. Equation (3) provides the following test statistic for the MEWMA control chart:

$$M_t = \lambda_1 X_t + (1-\lambda_1)M_{t-1} + (X_t - X_{t-1}); t = 1,2,... \quad (3)$$

Subsequently, in 2017, Khan *et al*. [4] further developed the control chart originally proposed by Patel and Divecha [3]. The test statistic of the MEWMA chart is given by Equation (4) as follows:

$$M_t = \lambda_1 X_t + (1-\lambda_1)M_{t-1} + \varsigma_1(X_t - X_{t-1}); t = 1,2,... \quad (4)$$

where λ_1 is a smoothing parameter of the EWMA control chart ($0 < \lambda_1 \le 1$), M_t is the MEWMA statistic, X_t is an observation from a process and ς_1 is a constant. Talordphop *et al*. [13], [14] also implemented the MEWMA mixed with moving average, which the performance outperformed when combined with the EWMA chart. The asymptotic control limits of the control chart are given by Equation (5),

$$UCL / LCL = \mu_0 \pm L_2 \sigma \sqrt{\frac{\lambda_1}{2-\lambda_1} + \frac{2\lambda_1(1-\lambda_1)}{2-\lambda_1}} \quad (5)$$

where μ_0 and σ are the mean and the standard deviation when the process is in control, respectively, and L_2 is the control coefficient of the MEWMA control chart.

2.1.3 Double Modified Exponentially Weighted Moving Average Control Chart (DMEWMA)

Proposed by Alevizakos *et al*. [5], this control chart is a combination of two MEWMA control charts. The DMEWMA statistic is defined by Equation (6) as follows:

$$\begin{cases} M_t = \lambda_1 X_t + (1-\lambda_1)M_{t-1} + \varsigma_1(X_t - X_{t-1}) \\ DM_t = \lambda_2 M_t + (1-\lambda_2)DM_{t-1} + \varsigma_2(M_t - M_{t-1}) \end{cases} \quad (6)$$

where λ_1, λ_2 is a smoothing parameter, $(0 < \lambda_1,\ \lambda_2 \le 1)$, DM_t is the DMEWMA statistic and ς_1, ς_2 are the additional parameters. The asymptotic control limits of the control chart are given by Equation (7)

$$UCL / LCL = \mu_0 \pm L_3 \sigma \sqrt{\sigma^2} \quad (7)$$

where μ_0 and σ are the mean and the standard deviation when the process is in control, respectively, and $L_3 > 0$ is the fit control width limit, σ^2 is the variance.

2.2 *Continuous distributions*

This study explores the use of the DMEWMA control chart in contexts involving continuous probability distributions, including the exponential and Weibull distributions.

2.2.1 Exponential distribution

With a parameter β, equal to the anticipated value (mean) of X, let the random variable X follow an exponential distribution. The following is the probability density function (PDF) for X [Equation (8)]

$$f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}} \quad ; \; x > 0, \; \beta > 0 \tag{8}$$

where β represents the scale parameter of the exponential distribution.

2.2.2 Weibull distribution

Let X be a random variable representing the lifetime. The Weibull distribution has parameters α and β. The probability density function (PDF) of X is defined as follows Equation (9):

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \quad ; \; x > 0, \; \alpha > 0, \; \beta > 0 \tag{9}$$

where α and β represent the shape and scale parameters of the Weibull distribution, respectively.

2.3 *Numerical Integral Equation (NIE) methods*

The methods for approximating the Average Run Length (ARL) of the DMEWMA control chart using Numerical Integration Equations are presented in this section. Let's $\psi(u)$ indicate the DMEWMA control chart's ARL. The following is the definition of the function:

$$ARL = \psi(u) = E_{\infty}(\bullet)$$

where τ_b is the stopping time, $E_{\infty}(\bullet)$ is the expectation under the assumption and b denotes the upper control limit (UCL) of the DMEWMA control chart. In this study, the Numerical Integration Equation (NIE) method is used to compute the ARL, as shown in Equation (10),

$$\psi(u) = 1 + \frac{1}{\lambda_1\lambda_2 + \varsigma_1\lambda_2 + \varsigma_2\lambda_1 + \varsigma_1\varsigma_2} \int_0^b \psi(y) f\left(\frac{y - (1-\lambda_2)u - (\lambda_2 - \lambda_1\lambda_2 - \lambda_1\varsigma_2)v + (\varsigma_1\lambda_2 + \varsigma_1\varsigma_2)e}{\lambda_1\lambda_2 + \varsigma_1\lambda_2 + \varsigma_2\lambda_1 + \varsigma_1\varsigma_2}\right) dy \tag{10}$$

where *v* and *e* denote the initial values when $v = M_0$ and $e = X_0$.

The quadrature rule can be applied to approximate the integral in Equation (10) using a finite sum. A set of weights $\{w_j, j = 1, 2, ..., m\}$ and corresponding points $\{a_j, j = 1, 2, ..., m\}$ are given. The quadrature rule is then used to estimate the integral over the interval [0, b] as follows Equation (11):

$$\int_0^b W(y) F(y) dy \approx \sum_{j=1}^{m} w_j F(a_j) \tag{11}$$

where *W(y)* and *f(y)* are given functions of quadrature rule. An approximation of the NIE method for function $\psi(u)$ is as follows:

$$\tilde{\psi}(u) = 1 + \frac{1}{\lambda_1\lambda_2 + \varsigma_1\lambda_2 + \varsigma_2\lambda_1 + \varsigma_1\varsigma_2} \sum_{j=1}^{m} w_j \tilde{\psi}(a_j) f\left(\frac{a_j - (1-\lambda_2)u - (\lambda_2 - \lambda_1\lambda_2 - \lambda_1\varsigma_2)v + (\varsigma_1\lambda_2 + \varsigma_1\varsigma_2)e}{\lambda_1\lambda_2 + \varsigma_1\lambda_2 + \varsigma_2\lambda_1 + \varsigma_1\varsigma_2}\right). \tag{12}$$

In this study, four approaches of the Numerical Integral Equations (NIE) method are employed, as outlined below Equation (13),

2.3.1 Gaussian rule method

Given,

$$f(A_j) = f\left(\frac{a_j - (1-\lambda_2)u - (\lambda_2 - \lambda_1\lambda_2 - \lambda_1\varsigma_2)v + (\varsigma_1\lambda_2 + \varsigma_1\varsigma_2)e}{\lambda_1\lambda_2 + \varsigma_1\lambda_2 + \varsigma_2\lambda_1 + \varsigma_1\varsigma_2}\right). \tag{13}$$

Form Equation (12), where $W(y)=1, -1<y<1.$ The Gaussian quadrature approximation for the Average Run Length (ARL) is given by the following expression:

$$\tilde{\psi}_G(u)=1+\frac{1}{\lambda_1\lambda_2+\varsigma_1\lambda_2+\varsigma_2\lambda_1+\varsigma_1\varsigma_2}\sum_{j=1}^{m}w_j\psi(a_j)f(A_j). \quad (14)$$

2.3.2 Midpoint rule method

The interval [0, b] is partitioned into *m* subintervals, each with a width of $h = b/m$. The approximation of the Average Run Length (ARL) using the midpoint rule is given as follows:

$$\tilde{\psi}_M(u)=1+\frac{1}{\lambda_1\lambda_2+\varsigma_1\lambda_2+\varsigma_2\lambda_1+\varsigma_1\varsigma_2}\sum_{j=1}^{m}w_j\psi(a_j)f(A_j) \quad (15)$$

where $a_j=w_j\left(j-\frac{1}{2}\right)$ and $w_j=\frac{b}{m}; j=1,2,...,m.$

2.3.3 Trapezoidal rule method

The interval [0, b] is partitioned into m subintervals, each with a width of $h = b/m$. The approximation of the Average Run Length (ARL) using the Trapezoidal Rule is given as follows:

$$\tilde{\psi}_T(u)=1+\frac{1}{\lambda_1\lambda_2+\varsigma_1\lambda_2+\varsigma_2\lambda_1+\varsigma_1\varsigma_2}\sum_{j=1}^{m}w_j\psi(a_j)f(A_j) \quad (16)$$

where $a_j=w_jj$ and $w_j=\frac{b}{m}; j=1,2,...,m-1$ in other cases, $w_j=\frac{b}{2m}.$

2.3.4 Simpson's rule method

The interval [0, b] is partitioned into 2m subintervals, each with a width of $h = b/2m$. The approximation of the Average Run Length (ARL) using the Simpson's Rule is given as follows:

$$\tilde{\psi}_S(u)=1+\frac{1}{\lambda_1\lambda_2+\varsigma_1\lambda_2+\varsigma_2\lambda_1+\varsigma_1\varsigma_2}\sum_{j=1}^{m}w_j\psi(a_j)f(A_j) \quad (17)$$

where $a_j=w_jj$, $w_j=\frac{4}{3}\left(\frac{b}{2m}\right); j=1,3,...,2m-1$ and $w_j=\frac{2}{3}\left(\frac{b}{2m}\right); j=2,4,...,2m-2,$ in other case, $w_j=\frac{1}{3}\left(\frac{b}{2m}\right).$

3 Results and Discussion

3.1 *Numerical study*

This study investigates and compares methods for approximating the Average Run Length (ARL) using the Numerical Integral Equations (NIE) method for the DMEWMA control chart when the underlying data follow exponential and Weibull distributions. The comparison is based on CPU processing times. The in-control ARL, denoted as ARL_0, is set to 370. The weighting parameters of the DMEWMA control chart are set to $\lambda_1 = 0.2$ and $\lambda_2 = 0.1$. The levels of mean shift in the process are defined as 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, and 1, respectively. In addition, the performance of the DMEWMA control chart is compared with that of the MEWMA and EWMA control charts based on the out-of-control Average Run Length (ARL_1).

Average Extra Quadratic Loss (AEQL) is a method used to measure the average additional loss when a production or quality control process goes out of control. It is based on the quadratic loss function, which accounts for how far observed values deviate from the target. In this study, AEQL is used to compare the performance of the DMEWMA control chart with the MEWMA and EWMA charts. AEQL is calculated as follows:

$$AEQL=\frac{1}{\Delta}\sum_{\delta_i=\delta_{\min}}^{\delta_{\max}}\left(\delta_i^2\times ARL(\delta_i)\right)$$

where δ represents the particular change in the process, and Δ represents the sum of the number of division from $\delta_{\min}$ to $\delta_{\max}$. In this study, $\Delta = 8$ is determined from $\delta_{\min} = 0.01$ to $\delta_{\max} = 2.00$. The control chart with the lowest AEQL values performs the best. The following describes the ARL calculation method used for performance comparison:

Step 1: Parameter determination for the control chart and the underlying distribution:

• shape (α) and scale (β) parameters of the exponential and Weibull distributions.

• smoothing parameters λ_1 and λ_2 for DMEWMA control chart.

• set the acceptable in-control ARL_0 to 370 and specify the shift size (δ).

Step 2: Calculate the UCL that corresponds to the desired ARL for the control process.

Step 3: Approximate ARL using the NIE approach by using Equations (14)–(17).

Step 4: Comparative evaluation of the performance of DMEWMA with MEWMA and EWMA control charts.

Table 1 presents the approximation of the Average Run Length (ARL) using the Numerical Integration Estimation (NIE) method for the DMEWMA control chart. The results indicate that the ARL values do not differ significantly across various levels of shifts. However, when considering computational efficiency, both the midpoint rule and the trapezoidal rule exhibit similarly low CPU processing times. Nonetheless, a detailed analysis at each individual shift level reveals that the midpoint rule consistently achieves the lowest CPU time. Consequently, the midpoint rule was selected for comparison with other control charts in the subsequent tables.

Tables 2 and 3 compare the ARL and AEQL performance of the DMEWMA, MEWMA, and EWMA control charts using the midpoint rule under an exponential distribution. The results show that DMEWMA outperforms MEWMA and EWMA control charts at all shift sizes. Additionally, for AEQL, the exponential distributions with parameters $\beta = 2$ and $\beta = 5$ yields the highest detection performance when $\varsigma = -\lambda / 2$ and $\varsigma = -\lambda / 4$, respectively.

Tables 4 and 5 apply the Midpoint rule under a Weibull distribution to compare the performance of the DMEWMA, MEWMA, and EWMA control charts in terms of ARL and AEQL. The results show that the DMEWMA chart achieves the best shift detection performance when $\varsigma = -\lambda / 4$.

Table 1: The ARL values of the DMEWMA control chart when given $\lambda_1 = 0.2$, $\lambda_2 = 0.1$ and *m* =500.

Distributions and Methods		δ								
		0	**0.01**	**0.03**	**0.05**	**0.1**	**0.3**	**0.5**	**1**	**2**
Exponential(2)	$\tilde{L}_M(u)$	370 **(1.641)**	363.584 **(1.709)**	351.369 **(1.609)**	339.913 **(1.625)**	314.176 (1.656)	240.374 **(1.625)**	194.139 (1.704)	130.731 **(1.704)**	79.178 (1.640)
	$\tilde{L}_G(u)$	370 (6.547)	363.584 (6.540)	351.369 (6.375)	339.913 (6.422)	314.176 (6.321)	240.374 (6.407)	194.139 (6.390)	130.731 (6.546)	79.178 (6.340)
	$\tilde{L}_T(u)$	370 (1.657)	363.584 (1.784)	351.369 (1.671)	339.913 (1.688)	314.176 **(1.625)**	240.374 (1.656)	194.139 **(1.640)**	130.731 (1.709)	79.178 **(1.525)**
	$\tilde{L}_S(u)$	370 (6.500)	363.584 (6.525)	351.369 (6.531)	339.913 (6.563)	314.176 (6.531)	240.374 (6.500)	194.139 (6.531)	130.731 (6.525)	79.178 (6.390)
Exponential(5)	$\tilde{L}_M(u)$	370 (1.703)	365.496 **(1.656)**	356.820 **(1.672)**	348.560 **(1.656)**	329.545 (1.641)	271.002 (1.625)	230.647 **(1.641)**	169.147 **(1.525)**	111.884 **(1.672)**
	$\tilde{L}_G(u)$	370 (5.062)	365.496 (4.986)	356.820 (4.610)	348.560 (4.656)	329.545 (4.937)	271.002 (4.460)	230.647 (4.986)	169.147 (5.063)	111.884 (4.656)
	$\tilde{L}_T(u)$	370 **(1.659)**	365.496 (1.708)	356.820 (1.708)	348.560 (1.659)	329.545 (1.746)	271.002 **(1.525)**	230.647 (1.708)	169.147 (1.625)	111.884 (1.709)
	$\tilde{L}_S(u)$	370 (5.256)	365.496 (5.781)	356.820 (5.688)	348.560 (5.500)	329.545 (5.525)	271.002 (5.453)	230.647 (5.781)	169.147 (5.781)	111.884 (5.500)
Weibull(1,4)	$\tilde{L}_M(u)$	370 **(1.328)**	364.810 **(1.313)**	354.868 **(1.313)**	345.469 (1.297)	324.074 **(1.140)**	260.253 **(1.359)**	217.974 **(1.187)**	156.056 (1.422)	100.942 **(1.468)**
	$\tilde{L}_G(u)$	370 (4.563)	364.810 (4.859)	354.868 (4.610)	345.469 (4.953)	324.074 (4.656)	260.253 (5.063)	217.974 (4.937)	156.056 (4.516)	100.942 (4.969)
	$\tilde{L}_T(u)$	370 (1.406)	364.810 (1.547)	354.868 (1.750)	345.469 **(1.219)**	324.074 (1.609)	260.253 (1.578)	217.974 (1.703)	156.056 **(1.343)**	100.942 (1.563)

Table 1: *(continued)*.

Distributions and Methods		δ								
		0	0.01	0.03	0.05	0.1	0.3	0.5	1	2
	$\tilde{L}_S(u)$	370 (5.500)	364.810 (5.250)	354.868 (5.781)	345.469 (5.547)	324.074 (5.360)	260.253 (5.781)	217.974 (5.688)	156.056 (5.484)	100.942 (5.453)
Weibull(3,6)	$\tilde{L}_M(u)$	370 (1.422)	362.197 (1.312)	347.567 **(1.110)**	334.108 **(1.250)**	304.740 **(1.171)**	226.259 (1.282)	180.476 **(1.281)**	120.081 **(1.110)**	70.565 **(1.594)**
	$\tilde{L}_G(u)$	370 (5.594)	362.197 (5.937)	347.567 (4.672)	334.108 (5.250)	304.740 (5.297)	226.259 (5.516)	180.47 6(3.906)	120.081 (4.140)	70.565 (5.094)
	$\tilde{L}_T(u)$	370 **(1.312)**	362.197 **(1.251)**	347.567 (1.640)	334.108 (1.375)	304.740 (1.391)	226.259 **(1.140)**	180.476 (1.391)	120.081 (1.250)	70.565 (1.656)
	$\tilde{L}_S(u)$	370 (5.172)	362.197 (5.391)	347.567 (5.109)	334.10 8(6.344)	304.740 (6.157)	226.259 (6.016)	180.476 (5.531)	120.081 (5.844)	70.565 (6.469)

Note: The CPU times are in parentheses (unit: seconds).

Table 2: Efficiency comparison in terms of ARL1 and AEQL of the DMEWMA, MEWMA and EWMA control charts for exponential(2) when given $\lambda_1 = 0.2,\ \lambda_2 = 0.1$.

Control Chart		δ									AEQL
		0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	
EWMA		370 (0.922)	354.636 (0.937)	326.362 (0.922)	301.019 (0.906)	248.266 (0.937)	129.165 (0.953)	78.011 (0.953)	33.521 (0.969)	14.241 (0.907)	15.647
MEWMA with $\varsigma_1 = 1$		370 (1.063)	354.634 (1.064)	326.355 (1.078)	301.355 (1.015)	248.241 (1.031)	129.088 (1.062)	77.900 (1.032)	33.379 (1.062)	14.108 (1.032)	15.559
DMEWMA	$\varsigma_2 = 0.05$	370 (1.688)	337.961 (1.704)	286.191 (1.687)	246.284 (1.640)	178.091 (1.719)	71.575 (1.688)	38.408 (1.703)	13.692 (1.719)	4.545 (1.702)	6.325
	$\varsigma_2 = 0.5$	370 (1.671)	307.506 (1.688)	227.594 (1.687)	178.753 (1.703)	112.870 (1.687)	38.520 (1.719)	20.009 (1.687)	7.183 (1.703)	2.642 (1.719)	3.504
	$\varsigma_2 = 1$	370 (1.641)	363.584 (1.709)	351.369 (1.609)	339.913 (1.625)	314.176 (1.656)	240.374 (1.625)	194.139 (1.704)	130.731 (1.704)	79.178 (1.640)	65.244
	$\varsigma_2 = 2$	370 (1.656)	366.378 (1.687)	359.324 (1.625)	352.517 (1.703)	336.501 (1.641)	284.179 (1.656)	245.520 (1.687)	182.882 (1.657)	121.267 (1.672)	94.939
MEWMA with $\varsigma_1 = -\lambda/2$		370 (1.078)	340.101 (1.094)	290.861 (1.046)	252.113 (1.094)	184.311 (1.047)	74.699 (1.063)	39.941 (1.054)	14.128 (1.063)	4.684 (1.563)	6.543
DMEWMA	$\varsigma_2 = -\lambda/2$	370 (1.704)	**224.807** (1.553)	**84.552** (1.705)	**32.794** (1.641)	**4.077** (1.552)	**1.001** (1.688)	**1.000** (1.688)	**1.000** (1.740)	**1.000** (1.525)	**0.695**
	$\varsigma_2 = 0.05$	370 (1.625)	312.830 (1.709)	224.785 (1.740)	162.627 (1.656)	74.590 (1.500)	5.430 (1.703)	1.415 (1.624)	1.020 (1.709)	1.000 (1.626)	0.906
	$\varsigma_2 = 0.5$	370 (1.505)	354.009 (1.620)	324.477 (1.621)	297.894 (1.565)	242.233 (1.709)	115.523 (1.708)	61.845 (1.625)	18.714 (1.500)	4.573 (1.525)	8.295
	$\varsigma_2 = 1$	370 (1.640)	358.946 (1.709)	338.139 (1.526)	318.929 (1.625)	276.954 (1.525)	167.763 (1.709)	110.194 (1.500)	49.082 (1.708)	17.585 (1.641)	20.747
MEWMA with $\varsigma_1 = -\lambda/4$		370 (1.093)	305.546 (1.063)	224.762 (1.094)	176.259 (1.094)	111.820 (1.078)	40.138 (1.079)	22.146 (1.064)	9.095 (1.110)	3.805 (1.125)	4.407

Table 2: *(continued).*

Control Chart		δ = 0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	AEQL
DMEWMA	$\varsigma_2 = -\lambda/4$	370 (1.688)	296.061 (1.500)	190.897 (1.625)	124.259 (1.656)	44.355 (1.709)	2.039 (1.688)	1.045 (1.709)	1.001 (1.500)	1.000 (1.656)	0.800
	$\varsigma_2 = 0.05$	370 (1.704)	330.554 (1.526)	264.721 (1.500)	212.938 (1.516)	125.913 (1.704)	19.872 (1.709)	4.831 (1.708)	1.176 (1.625)	1.004 (1.525)	1.281
	$\varsigma_2 = 0.5$	370 (1.625)	358.163 (1.688)	335.927(1.704)	315.453 (1.500)	270.937 (1.688)	157.175 (1.687)	99.167 (1.500)	40.627 (1.626)	13.112 (1.526)	16.981
	$\varsigma_2 = 1$	370 (1.688)	358.375 (1.656)	336.726 (1.525)	316.997 (1.704)	274.696 (1.709)	168.912 (1.500)	114.607 (1.525)	56.063 (1.615)	23.257 (1.655)	24.603

Table 3: Efficiency comparison in terms of ARL_1 and AEQL of the DMEWMA, MEWMA and EWMA control charts for exponential(5) when given $\lambda_1 = 0.2,\ \lambda_2 = 0.1$.

Control Chart		δ = 0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	AEQL
EWMA		370 (0.969)	363.750 (0.953)	351.665 (0.953)	340.107 (0.891)	313.347 (0.953)	230.678 (0.969)	175.235 (0.922)	98.802 (0.937)	44.504 (0.954)	38.414
MEWMA with $\varsigma_1 = 1$		370 (1.047)	362.248 (1.062)	347.463 (1.047)	333.571 (1.063)	302.290 (1.016)	212.668 (1.047)	157.636 (1.063)	87.009 (1.079)	39.376 (1.063)	34.141
DMEWMA	$\varsigma_2 = 0.05$	370 (1.672)	341.219 (1.656)	295.019 (1.672)	259.567 (1.687)	198.911 (1.687)	100.329 (1.687)	65.482 (1.672)	33.396 (1.703)	15.639 (1.735)	13.810
	$\varsigma_2 = 0.5$	370 (1.640)	347.206 (1.641)	308.906 (1.609)	277.988 (1.672)	221.718 (1.641)	119.870 (1.687)	80.297 (1.672)	41.895 (1.705)	19.679 (1.704)	17.189
	$\varsigma_2 = 1$	370 (1.703)	365.496 (1.656)	356.820 (1.672)	348.560 (1.656)	329.545 (1.641)	271.002 (1.625)	230.647 (1.641)	169.147 (1.525)	111.884 (1.672)	78.140
	$\varsigma_2 = 2$	370 (1.734)	366.384 (1.672)	359.366 (1.656)	352.620 (1.687)	336.847 (1.672)	286.047 (1.672)	248.923 (1.672)	188.724 (1.688)	128.396 (1.734)	88.321
MEWMA with $\varsigma_1 = -\lambda/2$		370 (1.094)	299.859 (1.062)	217.078 (1.063)	169.836 (1.047)	109.496 (1.078)	44.128 (1.047)	27.053 (1.047)	13.289 (1.062)	6.376 (1.172)	5.697
DMEWMA	$\varsigma_2 = -\lambda/2$	370 (1.719)	297.731 (1.688)	193.382 (1.719)	126.155 (1.703)	44.355 (1.703)	1.762 (1.688)	1.018 (1.687)	1.000 (1.719)	1.000 (1.688)	0.708
	$\varsigma_2 = 0.05$	370 (1.703)	345.375 (1.719)	301.195 (1.703)	262.969 (1.688)	188.241 (1.688)	53.052 (1.719)	16.869 (1.565)	2.146 (1.734)	1.018 (1.735)	2.006
	$\varsigma_2 = 0.5$	370 (1.688)	349.786 (1.703)	314.692 (1.703)	285.288 (1.719)	229.221 (1.703)	119.525 (1.703)	74.931 (1.672)	32.851 (1.750)	11.692 (1.734)	12.493
	$\varsigma_2 = 1$	370 (1.719)	359.108 (1.641)	339.015 (1.609)	320.902 (1.688)	282.585 (1.687)	188.106 (1.671)	138.387 (1.688)	79.854 (1.672)	40.115 (1.688)	32.868
MEWMA with $\varsigma_1 = -\lambda/4$		370 (1.093)	272.419 (1.078)	178.259 (1.062)	132.409 (1.094)	80.496 (1.015)	31.188 (1.094)	19.288 (1.062)	9.889 (1.109)	5.137 (1.078)	4.377
DMEWMA	$\varsigma_2 = -\lambda/4$	370 (1.703)	**272.200** (1.703)	**148.033** (1.640)	**81.102** (1.734)	**18.916** (1.641)	**1.059** (1.672)	**1.000** (1.656)	**1.000** (1.640)	**1.000** (1.672)	**0.655**
	$\varsigma_2 = 0.05$	370 (1.703)	339.380 (1.703)	286.223 (1.688)	242.113 (1.672)	161.176 (1.671)	36.054 (1.687)	9.767 (1.672)	1.417 (1.687)	1.003 (1.703)	1.514

Table 3: *(continued).*

Control Chart		δ									AEQL
		0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	
DMEWMA	$\varsigma_2 = 0.5$	370 (1.640)	349.431 (1.735)	314.117 (1.687)	284.898 (1.672)	230.041 (1.703)	125.162 (1.703)	82.676 (1.672)	41.171 (1.719)	17.900 (1.735)	16.448
	$\varsigma_2 = 1$	370 (1.656)	354.554 (1.656)	327.138 (1.610)	303.555 (1.625)	256.885 (1.687)	157.436 (1.641)	112.275 (1.672)	63.901 (1.703)	33.062 (1.672)	26.894

Table 4: Efficiency comparison in terms of ARL_1 and AEQL of the DMEWMA, MEWMA and EWMA control charts for Weibull(1,4) when given $\lambda_1 = 0.2,\ \lambda_2 = 0.1$.

Control Chart		δ									AEQL
		0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	
EWMA		370 (1.125)	362.210 (1.141)	347.271 (1.125)	333.140 (1.110)	301.019 (1.094)	207.363 (1.109)	149.696 (1.063)	78.011 (1.093)	33.521 (1.125)	34.047
MEWMA with $\varsigma_1 = 1$		370 (1.203)	357.876 (1.188)	335.520 (1.218)	315.384 (1.188)	272.888 (1.235)	169.572 (1.187)	117.187 (1.203)	59.901 (1.219)	26.673 (1.218)	26.876
DMEWMA	$\varsigma_2 = 0.05$	370 (1.958)	351.533 (1.969)	319.450 (1.985)	292.537 (1.938)	241.064 (1.938)	138.864 (1.984)	95.637 (1.969)	51.746 (2.031)	25.423 (2.000)	24.164
	$\varsigma_2 = 0.5$	370 (1.953)	361.584 (1.985)	345.861 (2.015)	331.459 (2.016)	300.249 (1.984)	218.411 (1.985)	171.930 (1.984)	112.753 (2.031)	67.492 (2.015)	56.192
	$\varsigma_2 = 1$	370 (1.328)	364.810 (1.313)	354.868 (1.313)	345.469 (1.297)	324.074 (1.140)	260.253 (1.359)	217.974 (1.187)	156.056 (1.422)	100.942 (1.468)	80.275
	$\varsigma_2 = 2$	370 (2.015)	367.078 (1.953)	361.377 (1.984)	355.860 (1.984)	342.815 (1.969)	299.364 (1.969)	266.138 (1.938)	209.351 (2.016)	148.363 (2.000)	112.62
MEWMA with $\varsigma_1 = -\lambda/2$		370 (1.219)	306.914 (1.316)	228.234 (1.312)	181.136 (1.281)	118.511 (1.297)	47.514 (1.266)	28.551 (1.282)	13.378 (1.313)	6.041 (1.344)	6.354
DMEWMA	$\varsigma_2 = -\lambda/2$	370 (2.031)	302.885 (2.032)	203.657 (2.047)	137.578 (2.000)	52.784 (2.062)	2.339 (1.985)	1.048 (1.968)	1.000 (2.031)	1.000 (2.063)	0.819
	$\varsigma_2 = 0.05$	370 (2.078)	346.687 (2.031)	304.678 (2.031)	268.108 (2.047)	195.853 (2.047)	60.298 (2.031)	21.018 (2.063)	2.924 (2.063)	1.056 (2.046)	2.596
	$\varsigma_2 = 0.5$	370 (2.047)	358.937 (2.062)	338.263 (2.016)	319.333 (2.078)	278.416 (2.031)	173.533 (1.985)	117.680 (2.031)	55.405 (1.985)	20.154 (2.047)	23.123
	$\varsigma_2 = 1$	370 (1.984)	360.035 (2.000)	341.450 (2.031)	324.472 (2.000)	287.847 (1.969)	193.477 (1.984)	141.704 (2.016)	79.648 (2.047)	34.011 (2.031)	24.071
MEWMA with $\varsigma_1 = -\lambda/4$		370 (1.313)	289.777 (1.219)	201.821 (1.297)	154.591 (1.250)	97.138 (1.265)	38.274 (1.297)	23.446 (1.281)	11.656 (1.297)	4.774 (1.328)	5.203
DMEWMA	$\varsigma_2 = -\lambda/4$	370 (2.032)	**285.091** (1.984)	**170.045** (1.984)	**102.103** (2.000)	**29.586** (2.000)	**1.245** (2.032)	**1.003** (2.110)	**1.000** (2.032)	**1.000** (2.012)	**0.762**
	$\varsigma_2 = 0.05$	370 (2.031)	328.505 (2.000)	259.531 (2.047)	205.634 (2.016)	116.335 (1.985)	14.385 (1.953)	2.884 (2.047)	1.028 (2.063)	1.000 (2.015)	1.123
	$\varsigma_2 = 0.5$	370 (1.984)	354.646 (1.985)	327.080 (2.047)	303.047 (2.016)	254.638 (2.094)	148.384 (2.015)	99.653 (2.063)	49.181 (2.047)	20.488 (2.031)	21.629
	$\varsigma_2 = 1$	370 (2.000)	354.991 (2.031)	328.197 (1.9380)	304.988 (1.984)	258.621 (1.985)	157.992 (1.984)	111.647 (1.984)	61.963 (2.000)	30.834 (2.031)	28.889

Table 5: Efficiency comparison in terms of ARL₁ and AEQL of the DMEWMA, MEWMA and EWMA control charts for Weibull(3,6) when given $\lambda_1 = 0.2,\ \lambda_2 = 0.1$

Control Chart		δ 0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	AEQL
EWMA		370 (1.110)	354.265 (1.120)	325.293 (1.109)	299.315 (1.125)	245.262 (1.109)	124.199 (1.109)	73.603 (1.141)	31.657 (1.172)	14.499 (1.141)	15.345
MEWMA with $\varsigma_1 = 1$		370 (1.203)	354.219 (1.219)	325.163 (1.188)	299.112 (1.218)	244.910 (1.235)	123.556 (1.219)	72.873 (1.239)	30.915 (1.234)	13.839 (1.281)	14.892
DMEWMA	$\varsigma_2 = 0.05$	370 (2.031)	302.809 (1.985)	221.710 (2.000)	174.529 (2.016)	113.287 (2.008)	45.671 (1.985)	27.787 (2.016)	13.305 (2.031)	6.026 (2.041)	6.283
	$\varsigma_2 = 0.5$	370 (2.000)	354.950 (1.969)	328.258 (1.984)	305.313 (2.000)	259.933 (1.985)	163.125 (2.031)	118.823 (1.953)	70.371 (2.047)	37.627 (2.031)	33.62
	$\varsigma_2 = 1$	370 (1.422)	362.197 (1.312)	347.567 (1.110)	334.108 (1.250)	304.740 (1.171)	226.259 (1.282)	180.476 (1.281)	120.081 (1.110)	70.565 (1.594)	59.007
	$\varsigma_2 = 2$	370 (1.969)	366.197 (2.015)	358.831 (1.999)	351.767 (2.000)	355.313 (2.104)	282.787 (1.953)	244.641 (2.000)	182.364 (2.063)	117.473 (1.954)	92.957
MEWMA with $\varsigma_1 = -\lambda/2$		370 (1.266)	336.799 (1.312)	284.153 (1.281)	244.343 (1.297)	177.630 (1.296)	75.034 (1.328)	42.316 (1.281)	16.400 (1.360)	5.706 (1.359)	7.404
DMEWMA	$\varsigma_2 = -\lambda/2$	370 (2.031)	314.423 (2.110)	227.838 (2.110)	165.858 (2.063)	76.563 (1.985)	5.223 (2.031)	1.330 (2.000)	1.002 (2.000)	1.000 (2.014)	0.903
	$\varsigma_2 = 0.05$	370 (2.453)	319.504 (2.532)	238.973 (2.437)	179.471 (2.031)	89.329 (2.047)	7.553 (2.047)	1.658 (2.078)	1.006 (2.110)	1.000 (2.110)	0.961
	$\varsigma_2 = 0.5$	370 (2.041)	326.594 (2.047)	255.112 (2.047)	199.952 (2.063)	110.379 (20.31)	12.960 (2.016)	2.689 (2.063)	1.031 (2.062)	1.000 (2.109)	1.092
	$\varsigma_2 = 1$	370 (2.032)	328.188 (2.062)	258.836 (2.063)	204.803 (2.062)	115.687 (2.063)	14.667 (2.062)	3.081 (2.031)	1.044 (2.094)	1.000 (2.078)	1.134
MEWMA with $\varsigma_1 = -\lambda/4$		370 (1.326)	240.683 (1.329)	141.078 (1.328)	99.408 (1.313)	56.669 (1.296)	20.019 (1.312)	11.818 (1.308)	5.685 (1.343)	2.857 (1.359)	2.854
DMEWMA	$\varsigma_2 = -\lambda/4$	370 (2.048)	**218.247** (2.110)	**77.093** (2.000)	**28.021** (2.110)	**3.153** (2.047)	**1.001** (2.031)	**1.000** (2.031)	**1.000** (2.047)	**1.000** (2.100)	**0.691**
	$\varsigma_2 = 0.05$	370 (2.031)	324.302 (2.031)	249.817 (2.062)	193.135 (2.000)	103.127 (2.062)	10.860 (2.031)	2.248 (2.063)	1.018 (2.078)	1.000 (2.078)	1.041
	$\varsigma_2 = 0.5$	370 (2.047)	361.671 (2.032)	345.718 (2.047)	330.654 (2.015)	296.511 (2.078)	197.905 (2.047)	138.142 (2.047)	65.664 (2.078)	23.182 (2.110)	26.859
	$\varsigma_2 = 1$	370 (2.016)	359.190 (1.922)	339.040 (1.953)	320.645 (2.016)	281.033 (2.000)	179.853 (2.032)	125.516 (1.985)	63.087 (2.063)	25.149 (2.044)	26.900

3.2 *Real application*

In this part, actual data is used to test the suggested control chart. The Midpoint Rule approach is used to compare the MEMA, EWMA, and DMEWMA control charts' respective performances. The real-world dataset observations are an exponential distribution with parameter $\beta = 4.775$ and a Weibull distribution with parameter $\alpha = 3.01967$, and $\beta = 6.94815$. A test was conducted to determine whether the real data follow the exponential and Weibull distributions. The results are presented in Table 6.

Table 6: Testing the distribution of real data.

Data No.	Distribution	Parameter	*p*-value
1	Exponential	$\beta = 4.775$	0.436
2	Weibull	$\alpha = 3.01967, \beta = 6.94815$	0.609

The first real-world dataset concerns the average waiting time experienced by passengers due to flight delays on Hawaiian Airlines flights to Harry Reid International Airport in Las Vegas, Nevada [15]. The efficiency of the DMEWMA control chart is presented in Table 7 and Figure 1. The second real-world dataset pertains to the duration of additive manufacturing processes that influence the defect rate in the production process [16]. The efficiency of the DMEWMA control chart is presented in Table 8 and Figure 2.

Tables 7 and 8, comparison of the performance of the DMEWMA control chart with the MEWMA and EWMA control charts, using the Midpoint Rule, when the real-world dataset follows an exponential and a Weibull distribution. Based on AEQL and ARL_1 values, the DMEWMA chart demonstrates superior shift detection performance across all shift sizes.

Figure 1(a), the DMEWMA control chart detected the shift at the 11th to 30th observations. In Figure 1(b), the MEWMA control chart detected the shift at the 12th, 13th and 18th to 28th observations. In Figure 1(c), the EWMA control chart detected the shift at the 21th and 22th observations. Figure 2(a) the DMEWMA control chart detected the shift at the 7th, 9th, 10th,11th, 12th, 13th and 15th to 50th observations. In Figure 2(b), the MEWMA control chart detected the shift at the 18th, 19th, 22th, 30th, 36th, 47th, 48th and 49th observations. In Figure 2(c), the EWMA control chart shows that no observations are out of the control limit.

4 Discussion

The results of the current study, which involved Exponential and Weibull distributions, were found to be consistent with the performance of the DMEWMA control chart, as examined by Phuntuna [17], Neammai [18], and Sukparungsee [19]—all of whom focused on time series data, specifically the AR(p), MA(q), and SAR(p) models, respectively. In particular, when it came to identifying process shifts, the DMEWMA control chart outperformed the MEWMA and EWMA control charts. The superior performance of the DMEWMA chart lies in its ability to dynamically adjust parameters based on changing data patterns, unlike MEWMA and EWMA, which use fixed parameters. This leads to more efficient ARL values.

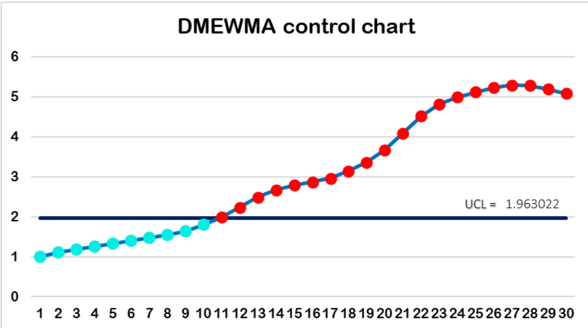
Additionally, the study by Neammai [11], which concentrated on the DMEWMA control chart, found that the Midpoint Rule was the most successful approach for estimating Average Run Length (ARL) utilizing four Numerical Integral Equation (NIE) approaches. Similarly, Paichit and Peerajit [20] investigated ARL estimation using the same four NIE methods applied to the CUSUM control chart under the Long Memory Seasonal Autoregressive Fractionally Integrated Moving Average (SARFIMA) model and found consistent results. The Midpoint Rule proved most efficient due to its simplicity, requiring fewer function evaluations and offering the shortest computation time.

Table 7: Efficiency comparison in terms of ARL_1 and AEQL of the DMEWMA, MEWMA and EWMA control charts for the real-world dataset observations are exponential distribution when given $\varsigma = -\lambda / 4$

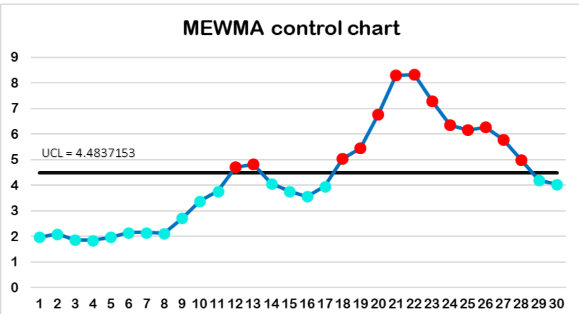
Control Chart	δ									AEQL
	0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	
DMEWMA	370 (1.718)	338.515 (1.688)	283.685 (1.703)	238.107 (1.704)	154.729 (1.672)	30.556 (1.688)	7.429 (1.687)	1.224 (1.687)	1.001 (1.719)	**1.533**
MEWMA	370 (1.047)	361.508 (1.031)	345.396 (1.094)	330.356 (1.078)	296.844 (1.110)	203.536 (1.078)	148.262 (1.047)	79.712 (1.078)	35.181 (1.062)	34.995
EWMA	370 (0.865)	363.459 (0.953)	350.831 (0.969)	338.78 (0.953)	310.972 (0.958)	226.017 (0.922)	169.973 (0.953)	94.284 (0.953)	41.994 (0.984)	41.175

Table 8: Efficiency comparison in terms of ARL_1 and AEQL of the DMEWMA, MEWMA and EWMA control charts for the real-world dataset observations are Weibull distribution when given $\varsigma_1 = 1, \varsigma_2 = 0.05$

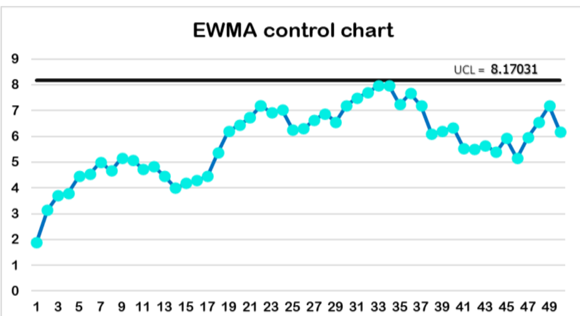
Control Chart	δ									AEQL
	0	0.01	0.03	0.05	0.1	0.3	0.5	1	2	
DMEWMA	370 (1.985)	304.138 (2.000)	223.886 (2.031)	176.811 (2.016)	115.279 (1.907)	46.781 (1.985)	28.568 (2.016)	13.795 (2.017)	6.353 (2.047)	**6.549**
MEWMA	370 (1.921)	333.225 (1.359)	275.468 (1.343)	232.372 (1.359)	162.188 (1.344)	63.365 (1.312)	35.776 (1.344)	15.654 (1.297)	7.203 (1.359)	7.699
EWMA	370 (1.100)	356.276 (1.141)	330.735 (1.125)	307.512 (1.125)	258.081 (1.240)	140.037 (1.110)	86.050 (1.141)	37.653 (1.140)	16.776 (1.141)	17.819



(a) Performance of DMEWMA chart

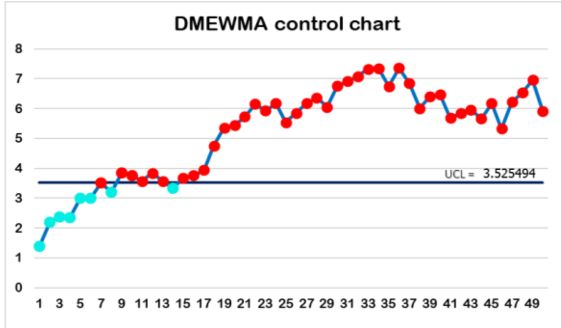


(b) Performance of MEWMA chart

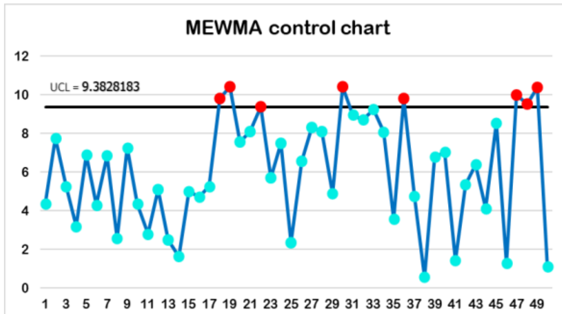


(c) Performance of EWMA chart

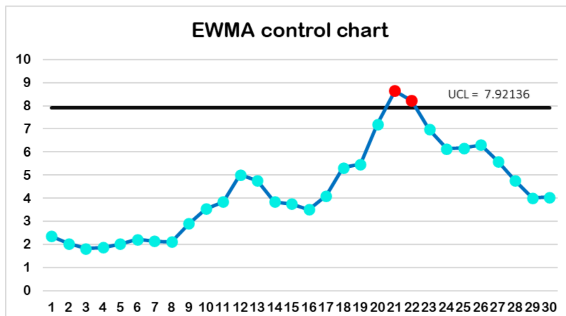
Figure 1: Control charts of the real-world dataset when observations are Exponential distribution.



(a) Performance of DMEWMA chart



(b) Performance of MEWMA chart



(c) Performance of EWMA chart

Figure 2: Control charts of the real-world dataset when observations are Weibull distribution.

5 Conclusions

The creation of control charts to enhance the identification of minute changes in process means, especially in the face of difficult data circumstances like autocorrelation and non-normality, has advanced significantly in recent years. By assessing the DMEWMA control chart's performance using numerical integration estimation (NIE) techniques and contrasting its efficacy under exponential and Weibull distributions, the current study adds to this continuing endeavor. In the context of the DMEWMA control chart. It is important to note that the underlying distribution of the data influences the nature of process changes; therefore, selecting appropriate parameters in accordance with these characteristics is crucial to achieving the lowest possible ARL_1 (i.e., the fastest possible detection). This study examined the approximation of ARL utilizing numerical integration estimation techniques. Evaluation of computational efficiency and performance assessment criteria were prioritized. The DMEWMA control chart is a good technique for tracking process changes, according to the study. Future research will look into expanding its use to include data from a larger variety of probability distributions, such as lognormal and gamma distributions.

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Author Contributions

Y.A.: conceptualization, investigation, reviewing and editing; S.W.: investigation, methodology, writing an original draft; S.S.: research design, data analysis; S.S.: conceptualization, data curation, writing—reviewing and editing, funding acquisition, project administration. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest

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